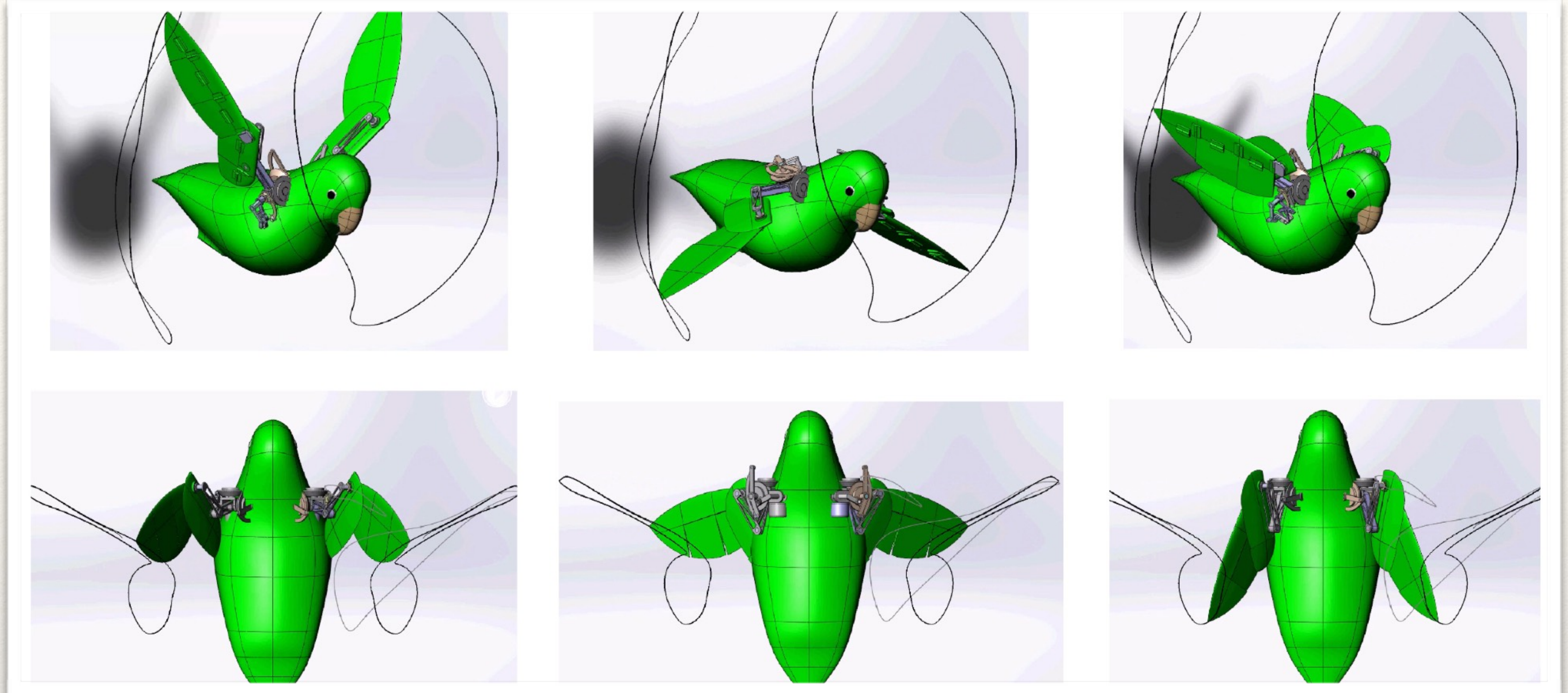


J. Michael McCarthy, University of California, Irvine

Design of Linkage Systems to Draw Specified Plane Curves

Polynomials, Kinematics and Robotics: A Conference Honoring Charles Wampler,
University of Notre Dame, June 5-7, 2017

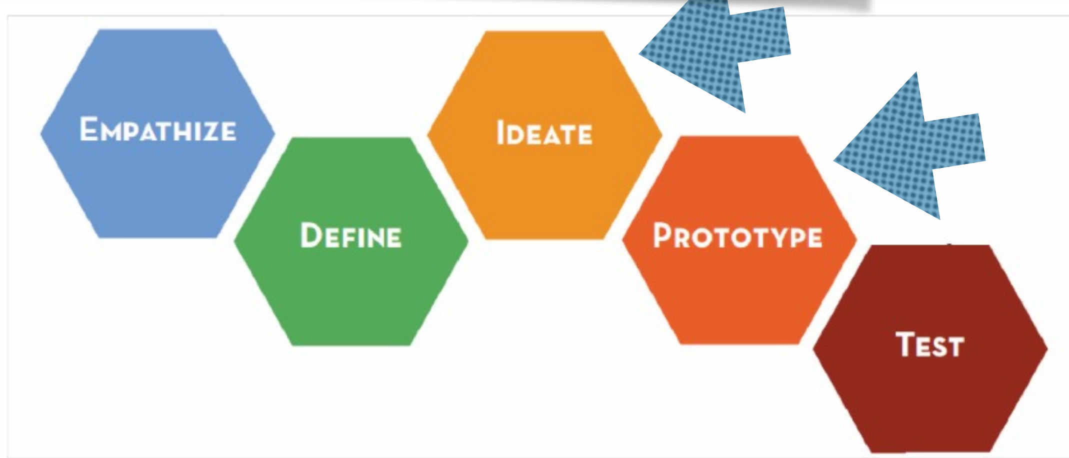
Kinematic Synthesis and Innovation

Kinematic synthesis of robotic systems is part of the design process.

*Massachusetts Institute of Technology, **Design Process***

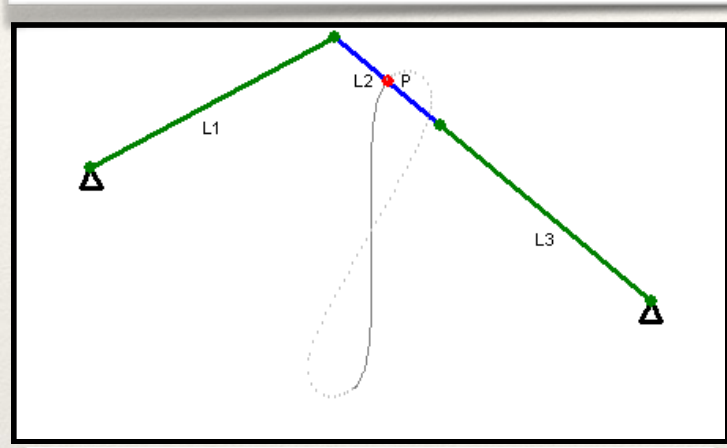
- 1) Identify Needs
  - What's the problem?
- 2) Information Phase
  - What exists?
- 3) Stakeholder Phase
  - What's wanted? And who wants it?
- 4) Planning/Operational Research
  - What's realistic? What limits us?
- 5) Hazard Analyses
  - What's safe? (What can go wrong?)
- 6) Specifications
  - What's required?
- 7) Creative Design
  - Ideation
- 8) Conceptual Design
  - Potential solutions
- 9) Prototype Design
  - Create a version of the preferred design
- 10) Verification
  - Does it work? If not, redesign

*Institute for Design at Stanford University, **Design Thinking***

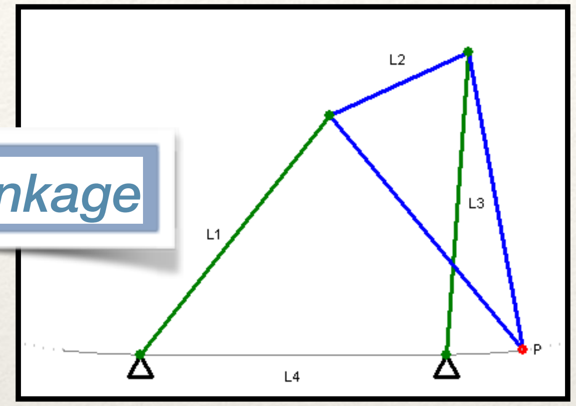


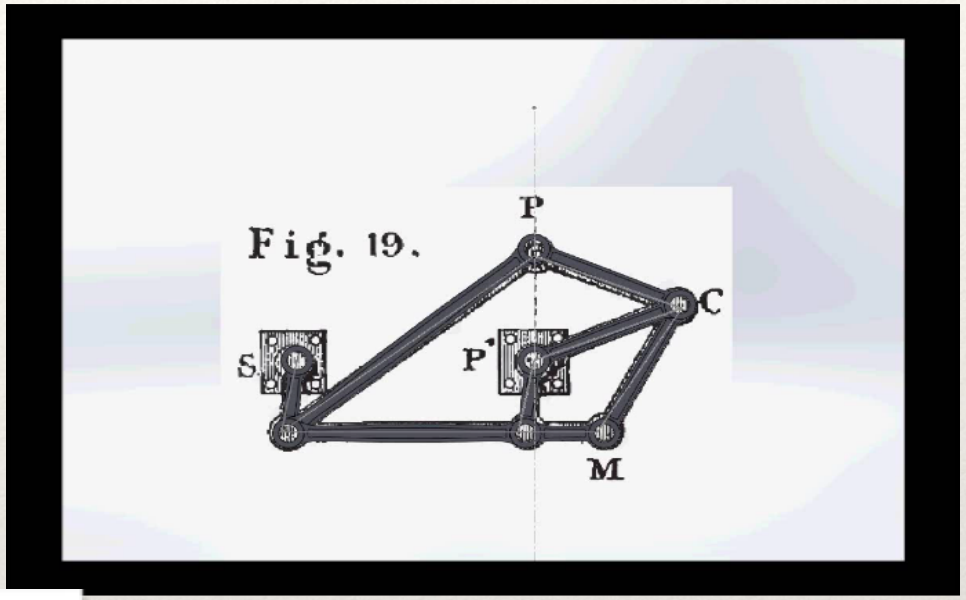
Drawing a Straight Line

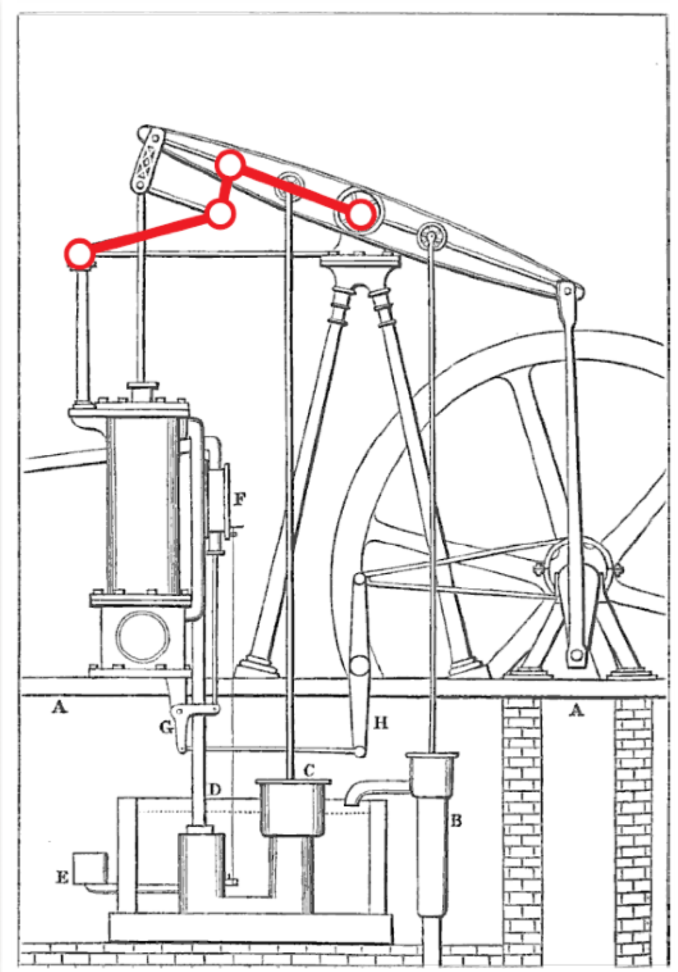
James Watt (1775) used a straight-line linkage to provide double-acting expansion in the steam engine.

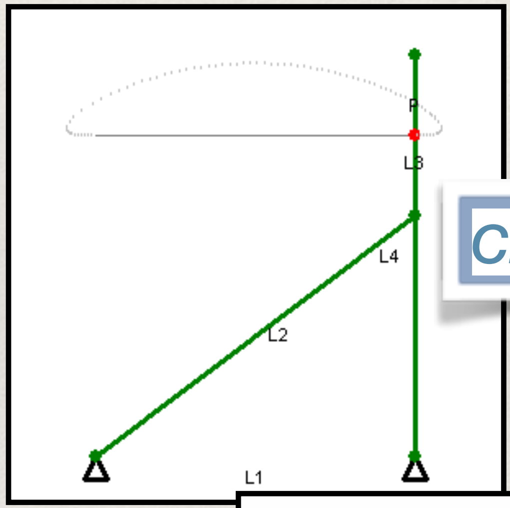


Roberts linkage



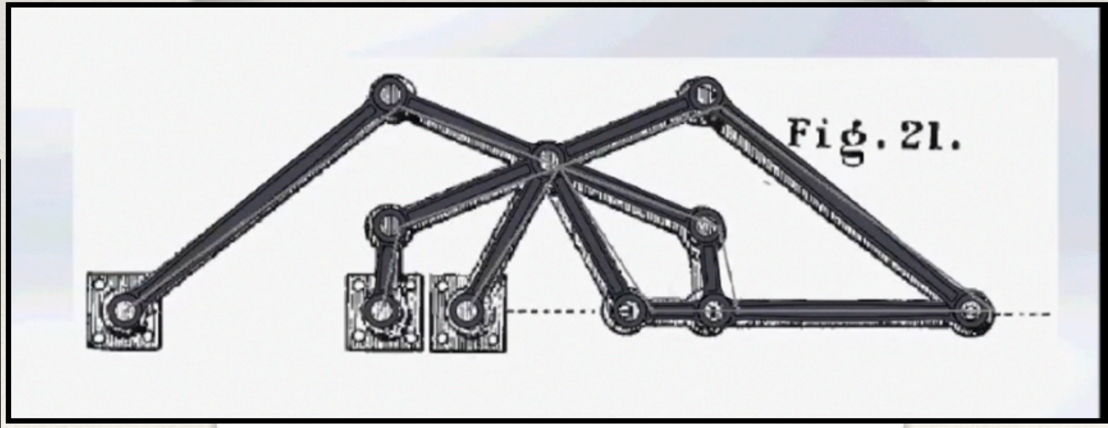


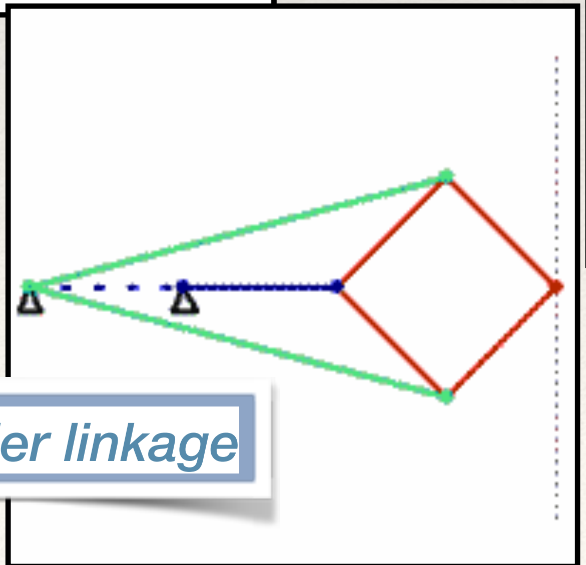




Chebyshev linkage

Kempe straight line linkage





Kempe rectilinear linkage

Peaucellier linkage

Drawing Algebraic Curves

Kempe (1876) showed for any algebraic curve there is linkage that draws the curve, Kempe's Universality Theorem.

On a General Method of describing Plane Curves of the n^{th} degree by Linkwork. By A. B. Kempe, B.A.

[*Read June 9th*, 1876.]

Lemma I.—*The Reversor.*—Let $O\xi\beta\alpha$ (Fig. 1) be the linkage known as the contra-parallelogram, $O\xi$ being equal to $\beta\alpha$, and $O\alpha$ to $\xi\beta$.

Make $\alpha\gamma$ a third proportional to $O\xi$ and $O\alpha$, and add the links $O\delta$, equal to $\alpha\gamma$, and $\delta\gamma$, equal to $O\alpha$.

Then the figure $O\alpha\gamma\delta$ is a contra-parallelogram similar to $O\xi\beta\alpha$; and the angle $\xi O\alpha$ is equal to the angle $\delta O\alpha$.

Thus, if $O\xi$ be made to make any angle with $O\alpha$, $O\delta$ will make the same angle with $O\alpha$ on the other side of it.*

negate

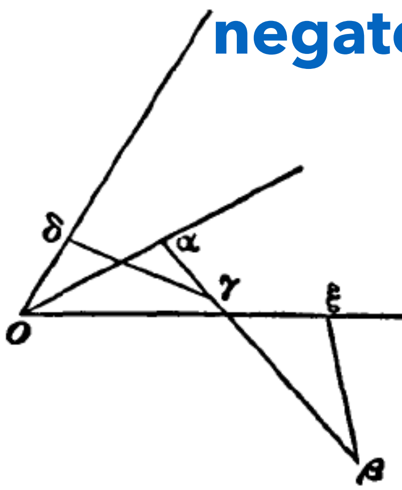


Fig. 1.

* This linkage, and the one next described, were first given by me in the senger of Mathematics," Vol. IV., pp. 122, 123, in a paper "On some new ages," §§ 4 and 8.

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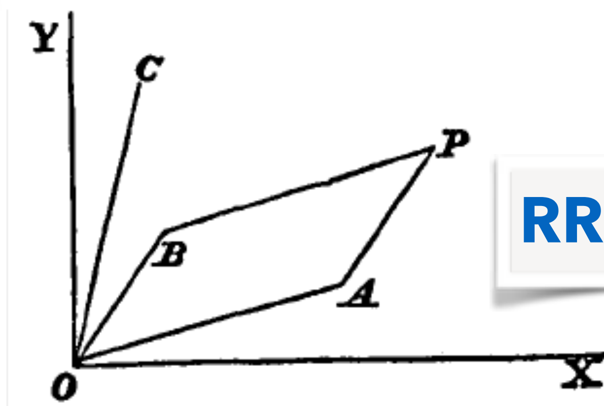

Fig. 5.

RR chain

$$f(x,y) = \sum[A\cos(r\phi \pm s\theta \pm a)] + C = 0$$

multiply

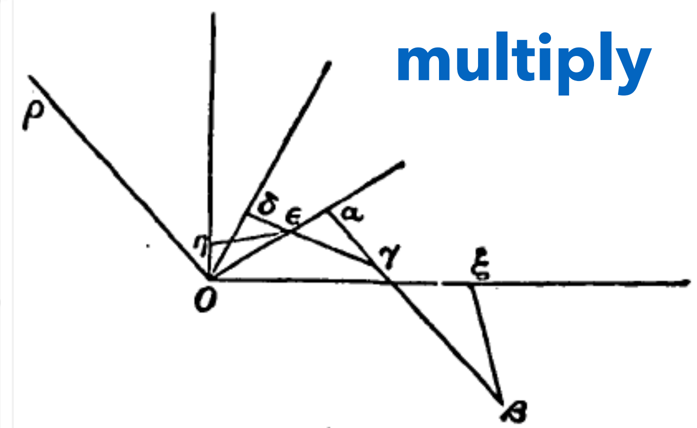

Fig. 2.

add

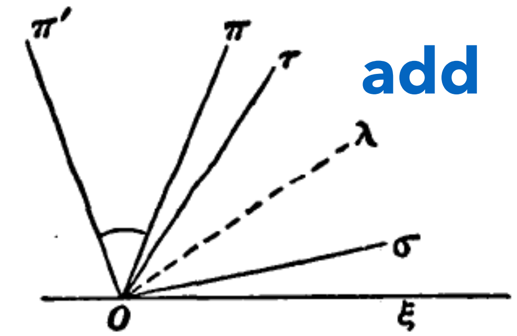

Fig. 3.

translate

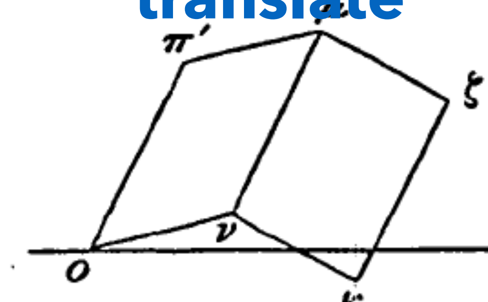

Fig. 4.

The method has, however, an interest, as showing that there is a way of drawing any given case; and the variety of methods of expressing

Kempe's Universality Theorem

*Mathematicians Michael Kapovich and John Millson provided what is recognized as the first complete proof of **Kempe's Universality Theorem***

PERGAMON

Topology 41 (2002) 1051–1107

TOPOLOGY

www.elsevier.com/locate/top

Universality theorems for configuration of planar linkages

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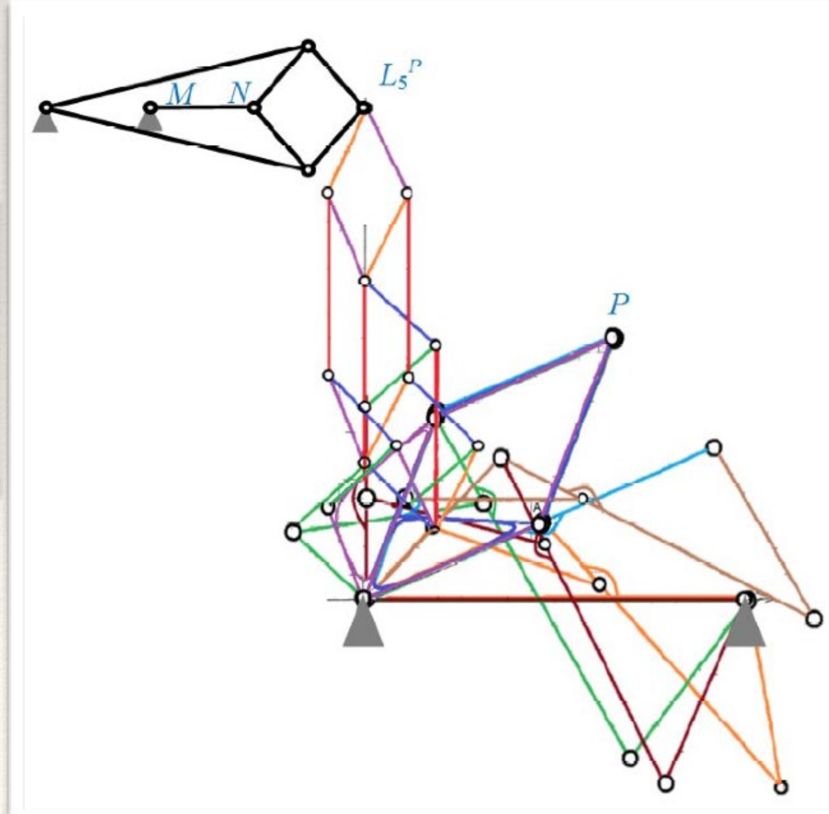
Received 7 September 2000; accepted 23 January 2001

Theorem 11.2. *Let $f = f(z,\bar{z})$, $f : \mathbb{C} \to \mathbb{R}$ be a polynomial function of the variables $z,\bar{z}$ and $\Gamma := f^{-1}(0) \subset \mathbb{C}$ be a real-algebraic curve. Pick an open (in the classical topology) bounded subset $U \subset \Gamma$. Then there is a closed $\mathbb{C}$-functional linkage $\mathscr{L}_0$ so that the input map $p_0 : C(\mathscr{L}_0, Z) \to \mathbb{C}$ is an analytically trivial polynomial covering over U.*

Saxena provides a useful demonstration of the design process to obtain a linkage consisting of 48 links and 70 joints to draw the curve:

$$C = (x-y)(x+y+2a) = 0$$



Saxena, A.: Kempe's linkages and the universality theorem. Resonance March, 220–237 (2011)

Freudenstein's Path Synthesis

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Synthesis of Path-Generating Mechanisms by Numerical Methods[1]

Algebraic methods in kinematic synthesis are extended to cases in which the development of iterative numerical procedures are required. Algorithms are developed for the numerical solution of nonlinear, simultaneous, algebraic equations. Convergence is obtained without the need for a "good" initial approximation.

The theory is applied to the nine-point path synthesis of geared five-bar motion, in terms of which four-bar motion may be considered as a special case.

Introduction

THE approximate synthesis of a given path by use of hinged mechanisms has been studied extensively in connection with four-bar mechanisms. Analytical [1][2] and graphical [2] solutions have been obtained for the problem specified in terms of five precision points and four crank angles; however, problems specified in terms of nine points (and no angles) have not been previously solved. Two published *formulations* of the nine-point path-synthesis problem are known to the authors [2, 3]. Both are for the four-bar mechanism; however, in the first no attempt is made to solve the equations, and in the second the suggested method of solution seems incomplete.

In this investigation we consider *geared five-bar mechanisms*, Fig. 1. Since they can generate a large variety of coupler curves [4, 5, 6], these linkages can be used for the solution of varied and complex design problems [7]. Their analysis is more involved than that of four-bar mechanisms, which can be considered as a special case of the geared five-bar—both mechanisms have equivalent coupler curves when the gear ratio is plus one [8, 9, 10, 11]. Previous geared five-bar path syntheses consist of a graphical-design procedure based on the two-degree-of-freedom property of the five-bar "loop" [12], and two analytical formulations of the prescribed crank-rotation problem [13, 19].

Four-bar linkages have (single) coupler links whose both hinge points describe a circular path. In contrast, five-bar linkages

parameters are eliminated at the start and the closure equations reduced to one (nonlinear) equation per precision condition [3]. Secondly, mathematical methods were developed in order to obtain convergence of the numerical iterations used in solving these equations. These mathematical methods, which are included in a digital computer program, contain the following new features:

1 The "bootstrap" procedure—this essentially eliminates the need for a "good" initial approximation.

2 The "position interchange" procedure—this reformulates the problem in order to eliminate the cause of nonconvergence.

3 The "quality-index-control" procedure—this assures convergence to solutions characterized by a reasonable ratio of maximum to minimum link length.

The Theory of Path Synthesis

Definition. Dimensional kinematic synthesis is the procedure of determining the dimensions of a mechanism from the required motion. When the synthesis is phrased in terms of generating a given curve, the procedure is called path synthesis.

Usually one does not attempt to generate the given curve exactly. In fact, only a limited class of motions could be so generated [18, 22], and in general it suffices if within a desired interval the generated curve is a good approximation to the given one. In this paper the approximate path-synthesis problem is formulated by specifying the location of the *precision points*

B. Roth and F. Freudenstein, **1963** "Synthesis of Path-Generating Mechanisms by Numerical Methods," ASME Journal of Engineering for Industry, 85:298-304, 1963.

IBM 7090 digital computer: $3m, 36bit, 32k core memory

C. W. Wampler

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Complete Solution of the Nine-Point Path Synthesis Problem for Four-Bar Linkages

The problem of finding all four-bar linkages whose coupler curve passes through nine prescribed points has been a longstanding unsolved problem in kinematics. Using a combination of classical elimination, multihomogeneous variables, and numerical polynomial continuation, we show that there are generically 1442 nondegenerate solutions along with their Roberts cognates, for a total of 4326 distinct solutions. Moreover, a computer algorithm that computes all solutions for any given nine points has been developed.

Introduction

The approximate synthesis of a given path by use of four-bar linkages has been studied extensively. Formulations in terms of four or five precision points along with specifications on crank angles or the position of the hinges of the mechanism have been solved (Freudenstein and Sandor, 1959; Shigley and Uicker, 1980; Erdman and Sandor, 1984; Morgan and Wampler, 1989; Subbian and Flugrad, 1989). However, the problem of finding four-bar linkages whose coupler curve passes through nine precision points, which was formulated as early as 1923 (Alt), has until now defied complete solution. Since nine general precision points is the largest number that can be prescribed, this formulation gives a designer maximum control

of degenerate solutions. By the theory of "parameter polynomial continuation" (Morgan and Sommese, 1989), we may ignore all the degenerate solutions and use only the nondegenerate ones as start points in subsequent continuations to find all nondegenerate solutions to any other problem of the class. Thus, we have not only established the generic number of nondegenerate solutions to the problem, but also have developed an efficient computer algorithm for finding them.

In any particular example, not all of the $1442 \times 3 = 4326$ solutions are useful. Most give linkages with complex link lengths, whereas others give real linkages that exhibit branch or order defects, or that have poor transmission angles, etc.

C. W. Wampler, A. J. Sommese, and A. P. Morgan, **1992**. "Complete solution of the nine-point path synthesis problem for four-bar linkages," Journal of Mechanical Design, 114(1):153–159.

or order defects, it is often difficult to find an acceptable solution by trial-and-error procedures. Only by finding all

The most concise formulation of the problem is obtained

The entire computational cost of the numerical reduction was 331.9 hours of CPU time on an IBM 3081. (The IBM 3081 is about 1/3 as fast as an IBM 3090.) Fortunately, that is a one-time only expense, and subsequent solutions of the problem cost only a small fraction as much.

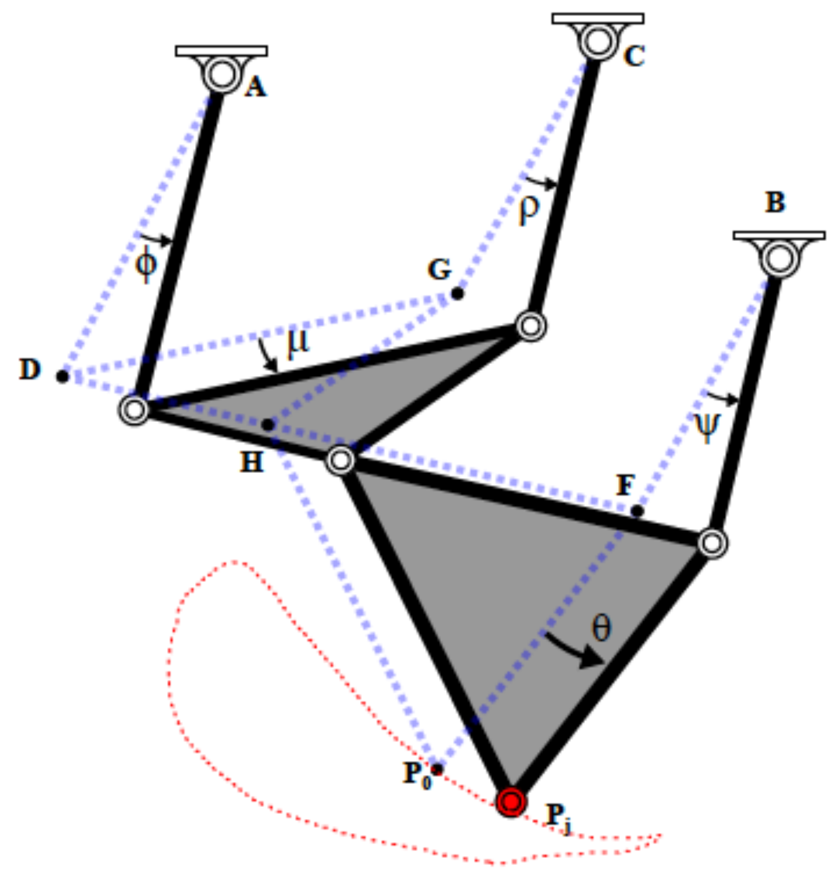
20 quadratic polynomials, total degree of $d=2^{20}=1{,}048{,}576$, multi-homogeneous degree $d=286{,}720$.
Today generic four-bar path synthesis is solved in minutes, and the parameter homotopy runs in seconds.

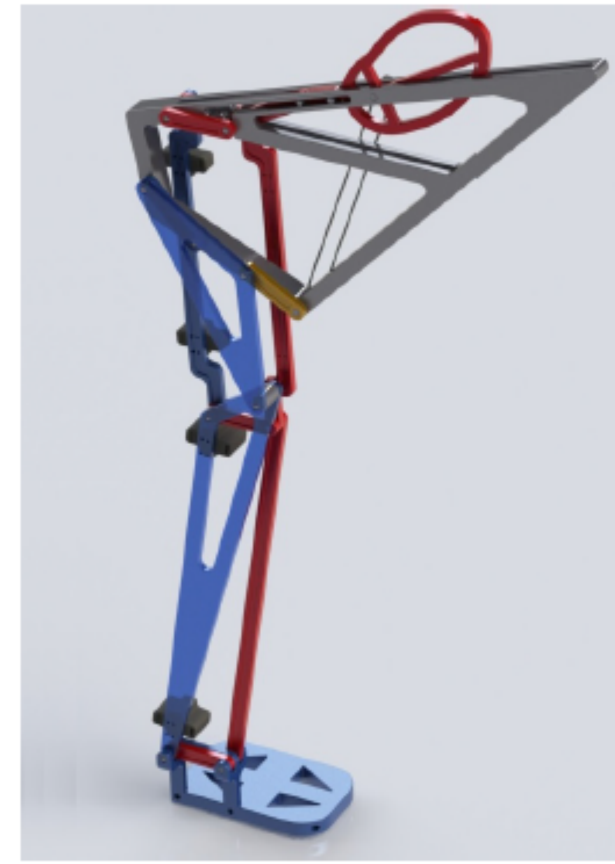
Unsolved: Six-bar Path Synthesis

N=15 points on a trajectory yields 154 quadratic equations in 154 unknowns.

Total degree $d=2^{154}$ or $d=2.3\mathrm{x}10^{46}$.

Beyond our current computation capabilities.





(b) Six-bar linkage guides a natural walking trajectory.

Normalization conditions and three pairs of loop equations:

$$\mathcal{A}_j: \begin{cases} Q_j(D-A)+U_j(H-D)+T_j(P_0-H)=P_j-A, \\ \bar{Q}_j(\bar{D}-\bar{A})+\bar{U}_j(\bar{H}-\bar{D})+\bar{T}_j(\bar{P}_0-\bar{H})=\bar{P}_j-\bar{A}, \end{cases} \quad j=1,\ldots,N-1,$$

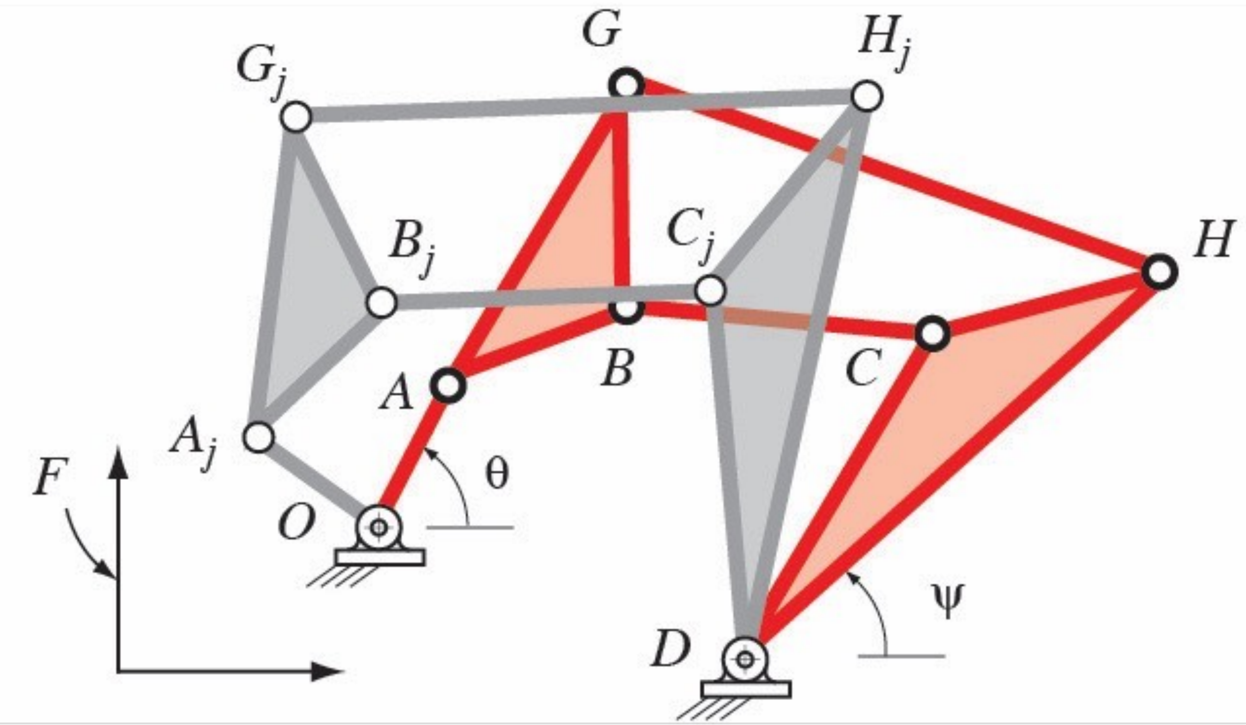
$$\mathcal{B}_j: \begin{cases} R_j(G-C)+U_j(H-G)+T_j(P_0-H)=P_j-C, \\ \bar{R}_j(\bar{G}-\bar{C})+\bar{U}_j(\bar{H}-\bar{G})+\bar{T}_j(\bar{P}_0-\bar{H})=\bar{P}_j-\bar{C}, \end{cases} \quad j=1,\ldots,N-1,$$

$$\mathcal{C}_j: \begin{cases} S_j(F-B)+T_j(P_0-F)=P_j-B, \\ \bar{S}_j(\bar{F}-\bar{B})+\bar{T}_j(\bar{P}_0-\bar{F})=\bar{P}_j-\bar{B}, \end{cases} \quad j=1,\ldots,N-1.$$

Solved: Six-bar Function Generation

For N=11 coordinated angles this yields 70 equations in 70 unknowns.
Bezout bound d = 2^{70} = 1.18×10^{21}, the
multi-homogeneous degree is d = 264×10^{6}.

300 hrs on 256 nodes of a high performance computing cluster.



Three sets of normalization equations,

$$R_j\bar{R}_j = 1, \quad S_j\bar{S}_j = 1, \quad T_j\bar{T}_j = 1, \quad j = 1, \ldots, N-1.$$

Two sets of loop equations:

$$\mathcal{A}_j : \begin{cases} (D + S_j(C - D)) - (O + Q_j(A - O) + R_j(B - A)) = T_j(C - B), \\ (\bar{D} + \bar{S}_j(\bar{C} - \bar{D})) - (\bar{O} + \bar{Q}_j(\bar{A} - \bar{O}) + \bar{R}_j(\bar{B} - \bar{A})) = \bar{T}_j(\bar{C} - \bar{B}), \end{cases} \quad j = 1, \ldots, N-1,$$

$$\mathcal{B}_j : \begin{cases} (D + S_j(H - D)) - (O + Q_j(A - O) + R_j((G - A)) = U_j(H - G), \\ (\bar{D} + \bar{S}_j(\bar{H} - \bar{D})) - (\bar{O} + \bar{Q}_j(\bar{A} - \bar{O}) + \bar{R}_j(\bar{G} - \bar{A})) = \bar{U}_j(\bar{H} - \bar{G}), \end{cases} \quad j = 1, \ldots, N-1.$$

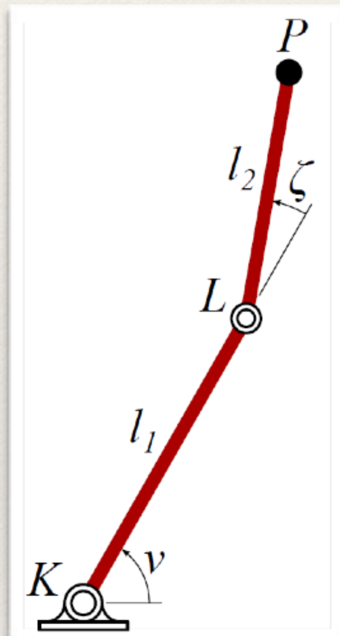
M. Plecnik and J. M. McCarthy, "Computational Design of Stephenson II Six-bar Function Generators for 11 Accuracy Points," ASME Journal of Mechanisms and Robotics, Vol 8(1), February 2016.

Modified Six-bar Path Synthesis

Path synthesis can be transformed to synthesis of a function generator

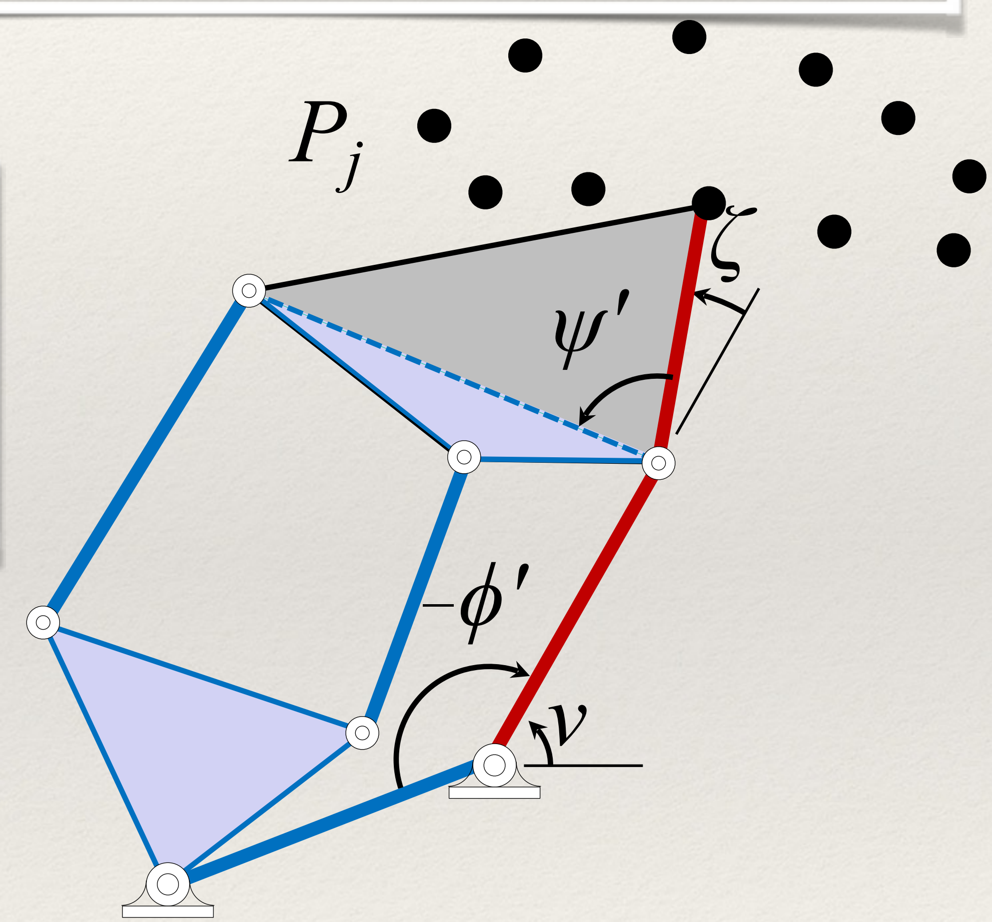
1. Specify an RR chain

2. Move the RR chain through 11 specified points



3. Compute the Inverse Kinematics of the RR chain to obtain the joint angle function

$(\nu_j, \zeta_j), j=0,\ldots,10$



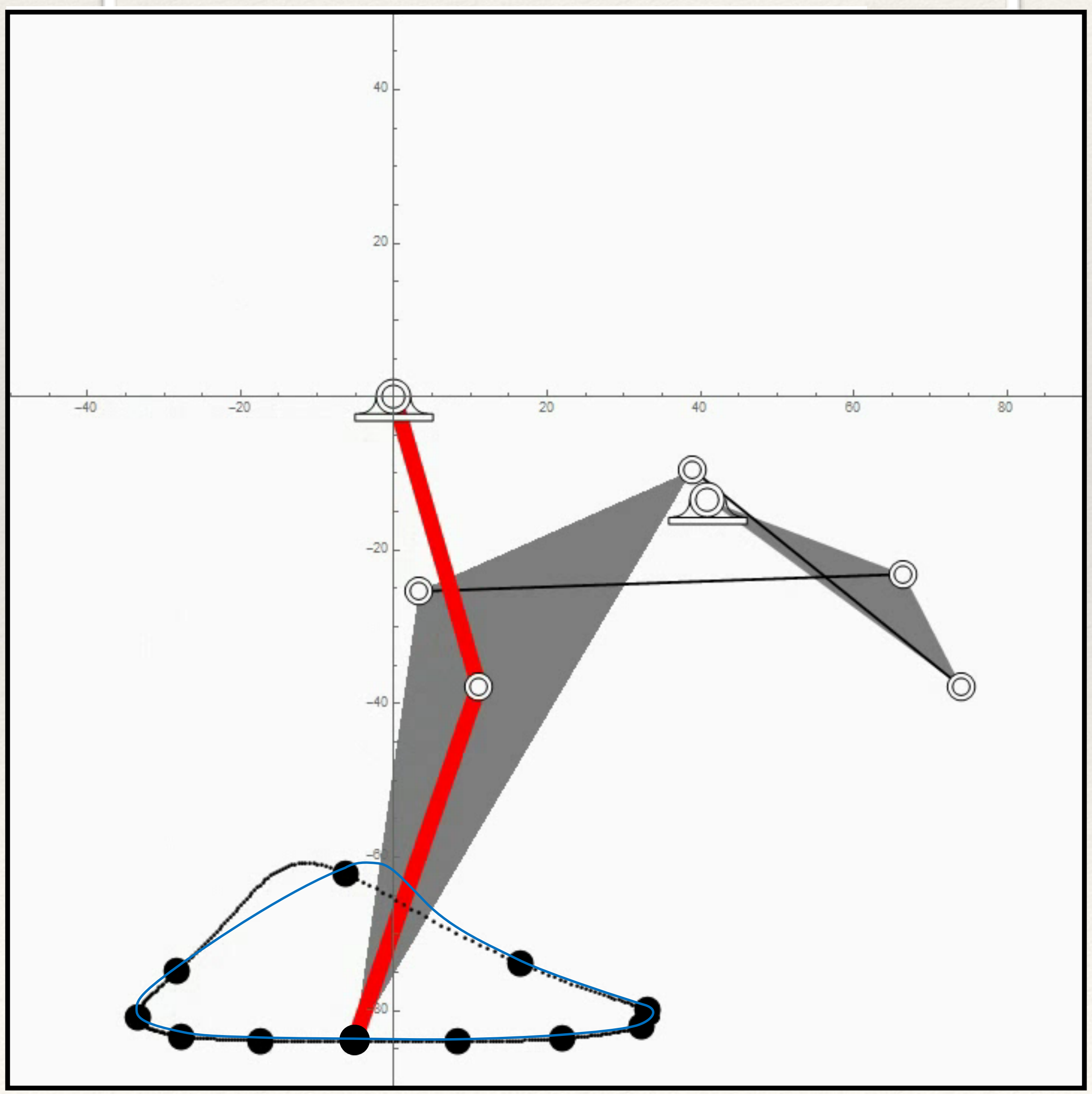
4. Solve the synthesis equations for 11 point Stephenson II function generators

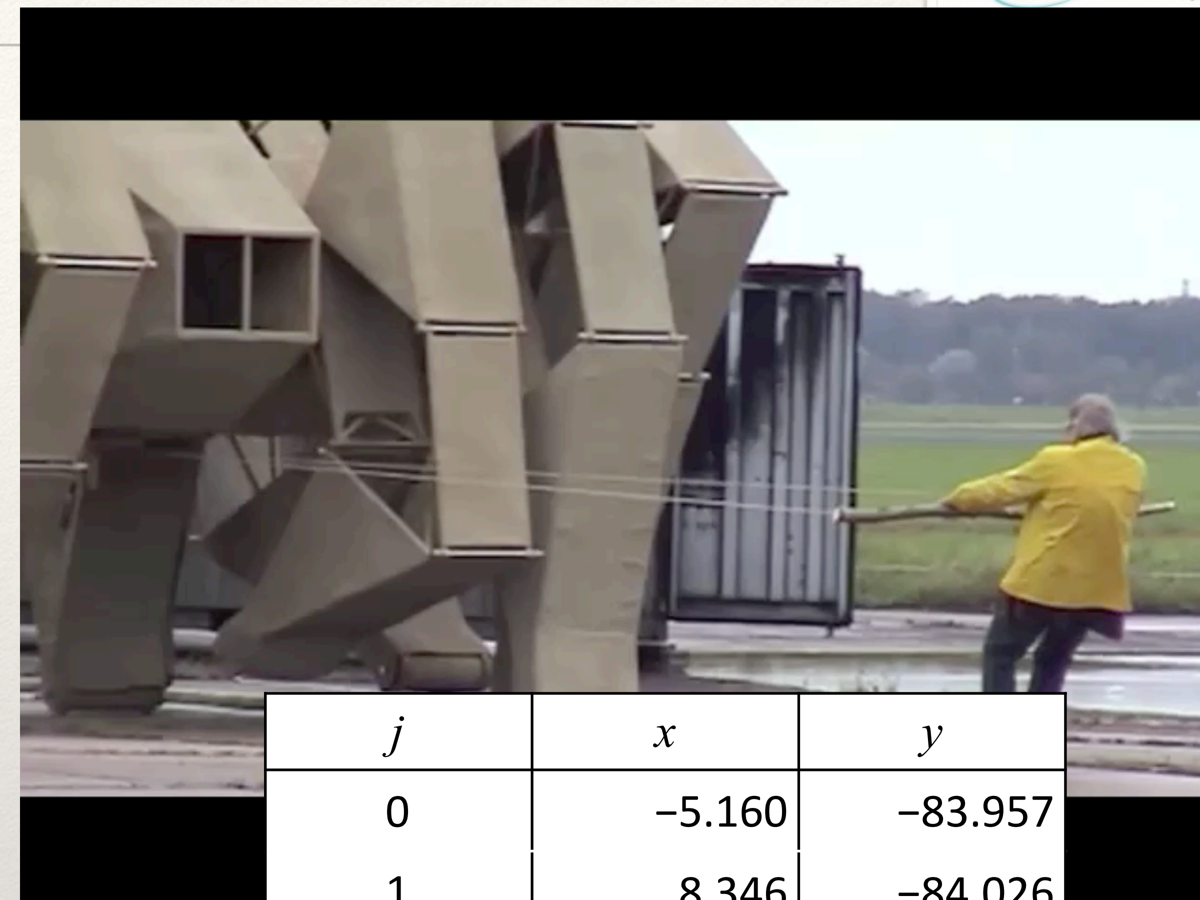
5. Attach the function generator to the RR chain

Walking Machines

Theo Jansen designed an eight-bar linkage for the legs of his Strandbeest

We can design a six-bar linkage with a similar walking gait



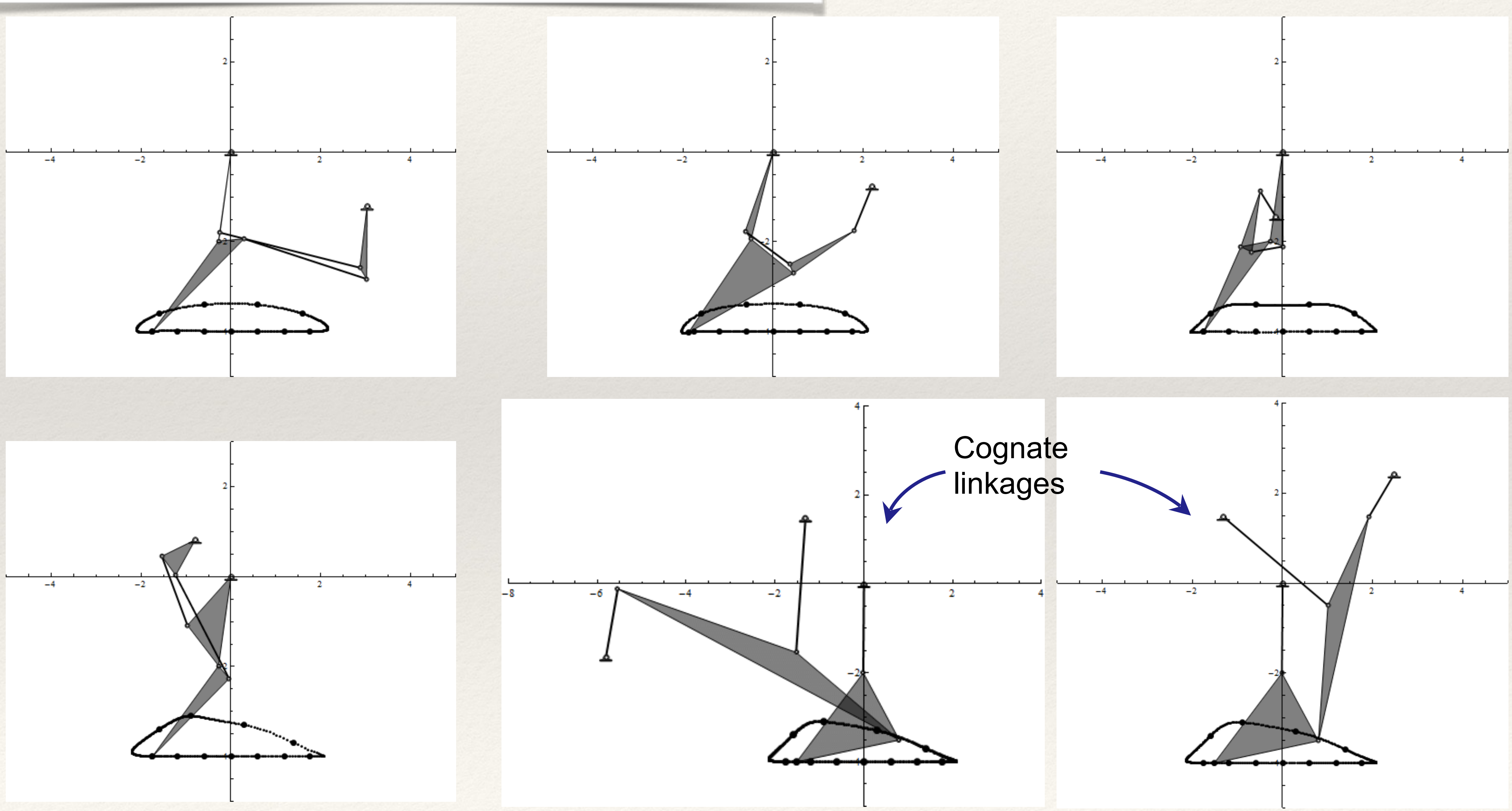


j	x	y
0	−5.160	−83.957
1	8.346	−84.026
2	21.993	−83.632
3	32.259	−82.128
4	33.018	−79.911
5	16.497	−73.889
6	−6.363	−62.120
7	−28.276	−74.865
8	−33.406	−80.964
9	−27.733	−83.440
10	−17.440	−84.032

Different Gaits

Once the general homotopy is solved, parameter homotopies execute rapidly.

Here are results for other foot trajectories

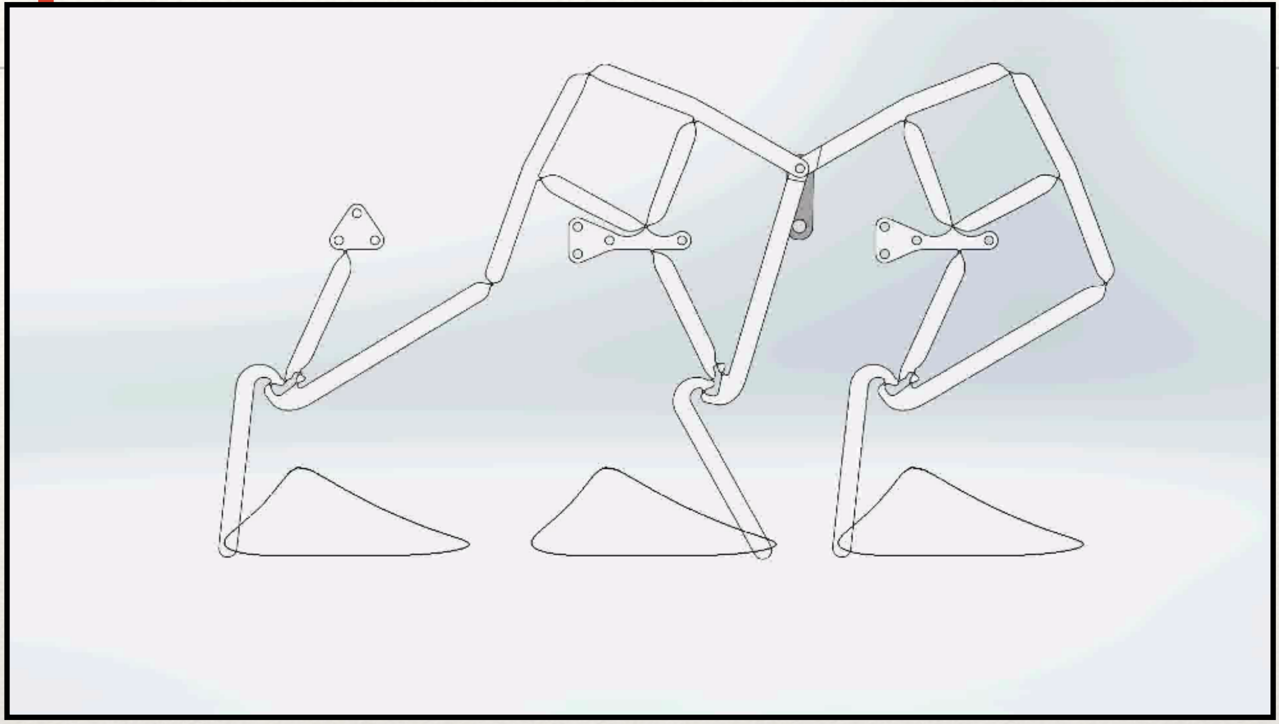


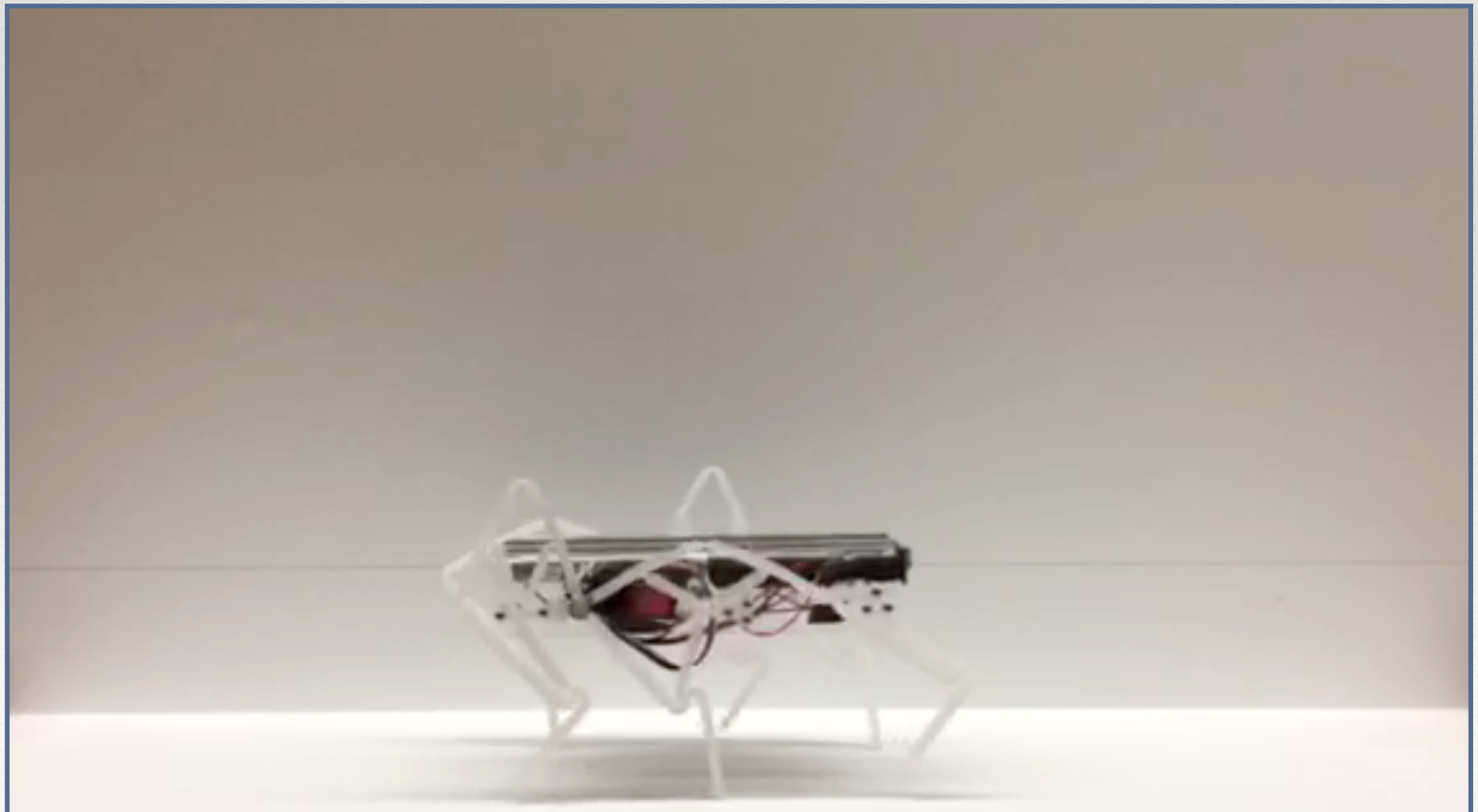
Prototype Walker

One of the leg designs was manufactured as a compliant linkage

Lasercut polypropylene

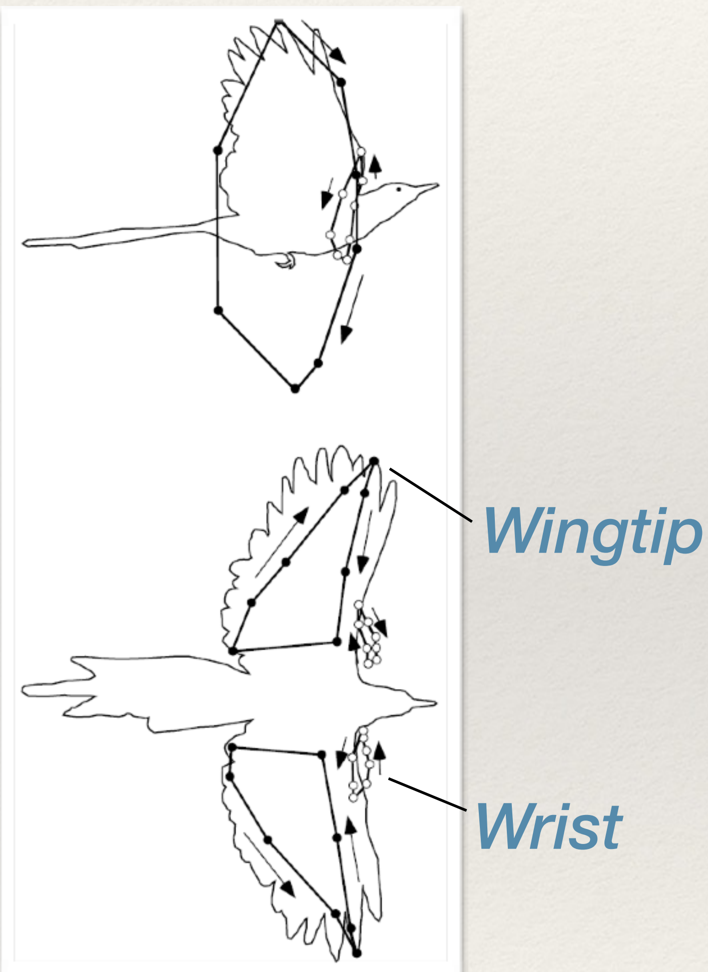
Used to build a robot about 30 cm in length





Wings Instead of Legs

Mechanically controlled serial chain that reproduces the wing-tip trajectory of the black-billed magpie.



Data obtained from Tobalske and Dial high speed video footage of a black-billed magpie flying

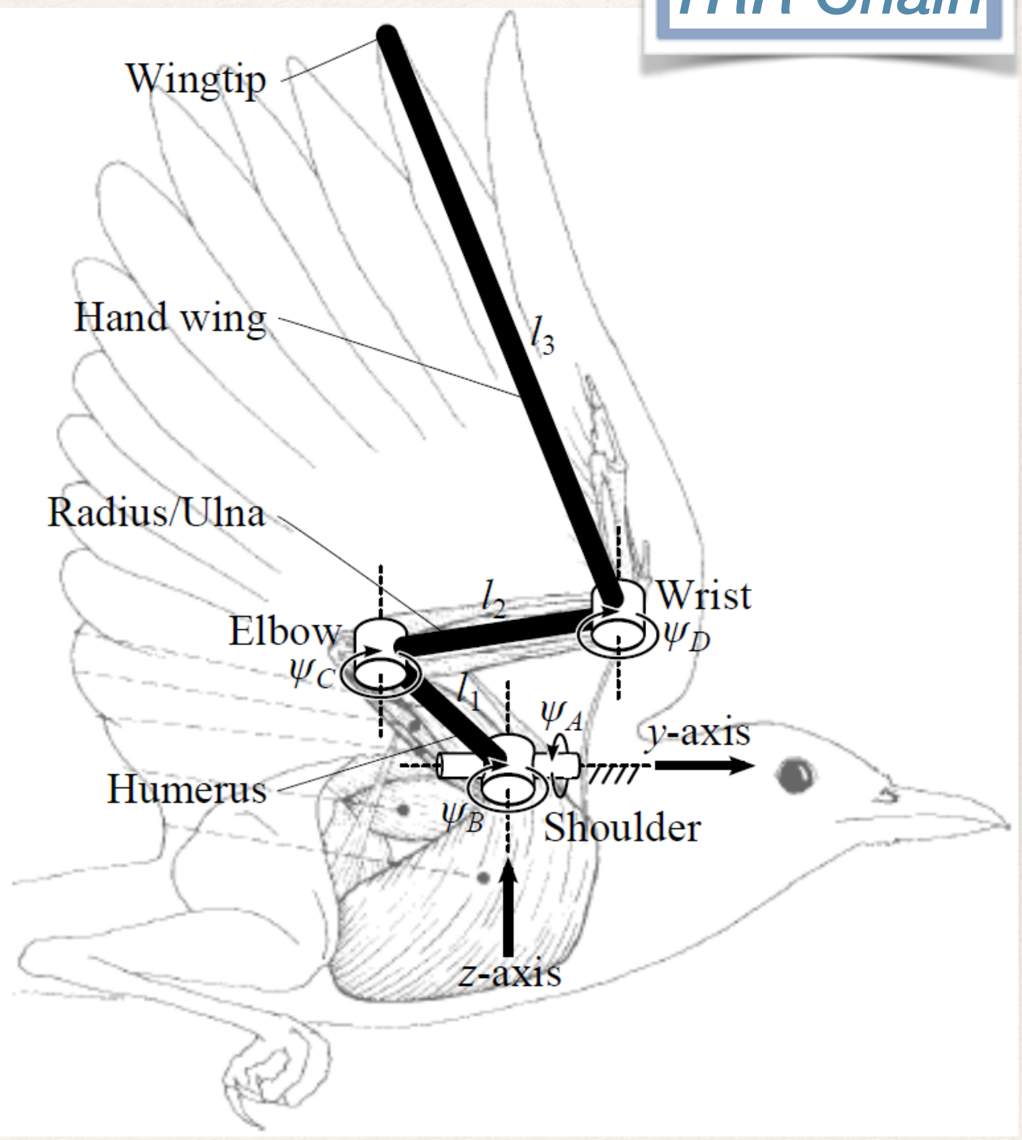
	Wrist	Wingtip
	{X, Y, Z}	{X, Y, Z}
1	{1.79, 0.80, 3.55}	{5.58, -2.87, 9.37}
2	{3.11, -0.05, 1.61}	{8.97, -0.04, 6.52}
3	{3.37, -0.61, -0.24}	{10.77, 0.68, 2.26}
4	{2.98, -0.30, -1.18}	{9.63, 0.68, -1.10}
5	{2.10, 0.14, -1.40}	{5.49, -1.08, -6.33}
6	{1.48, 0.26, -0.46}	{1.66, -2.14, -7.49}
7	{0.91, 0.48, 1.08}	{1.35, -5.62, -3.92}
8	{0.57, 0.89, 2.24}	{2.67, -5.66, 3.42}

B. Tobalske, and K. Dial, 1996. "Flight kinematics of black-billed magpies and pigeons over a wide range of speeds," The Journal of Experimental Biology, 199(2):263–280.

Mechanically Driven Spatial Chain

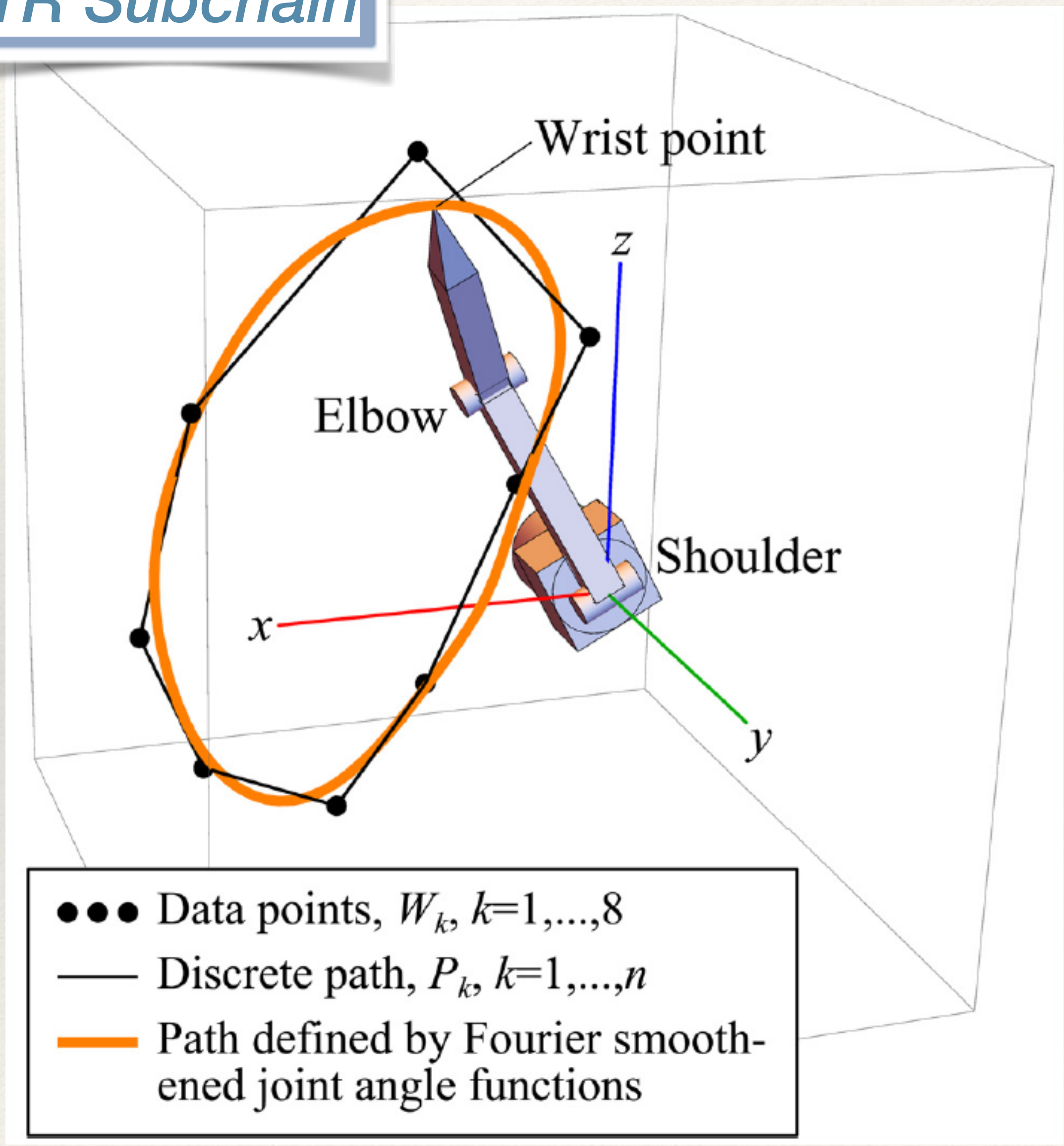
A TRR spatial chain was selected to model the magpie wing

TRR Chain



$$\hat{D}(\psi_A, \psi_B, \psi_C, \psi_D) = \hat{X}\left(-\frac{\pi}{2}, 0\right) \hat{Z}(\psi_A, 0)\, \hat{X}\left(\frac{\pi}{2}, 0\right) \hat{Z}(\psi_B, 0) \times \hat{X}(0, l_1)\, \hat{Z}(\psi_C - \psi_B, 0)\, \hat{X}(0, l_2)\, \hat{Z}(\psi_D - \psi_C, 0)\, \hat{X}(0, l_3)$$

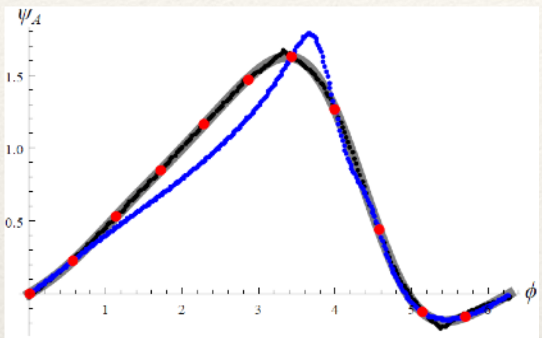
TR Subchain



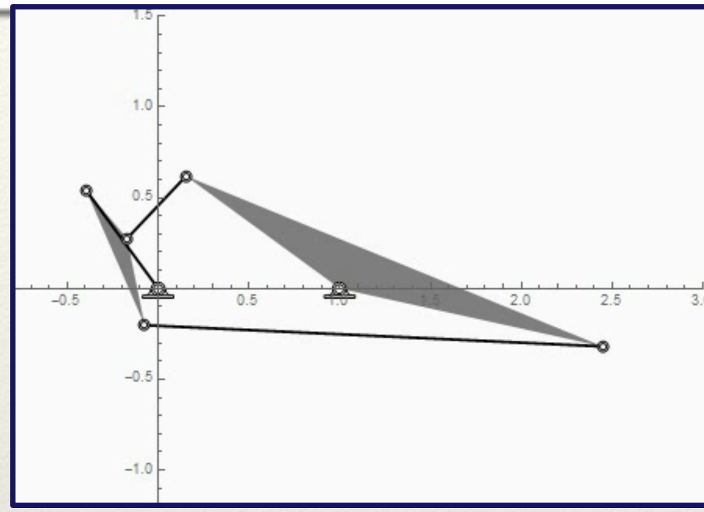
$$\hat{D}_{TR}(\psi_A, \psi_B, \psi_C) = \hat{X}(-\frac{\pi}{2}, 0)\hat{Z}(\psi_A, 0)\hat{X}(\frac{\pi}{2}, 0)\hat{Z}(\psi_B, 0) \times \hat{X}(0, l_1)\hat{Z}(\psi_C - \psi_B, 0)\hat{X}(0, l_2)$$

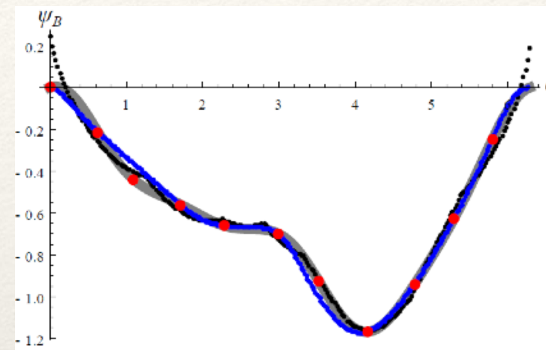
Driving Six-bar Linkages

The four target functions and the six-bar linkages that generate them

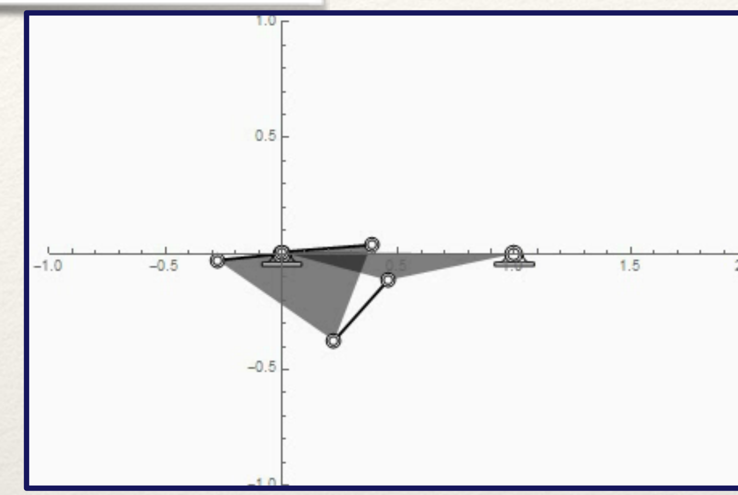


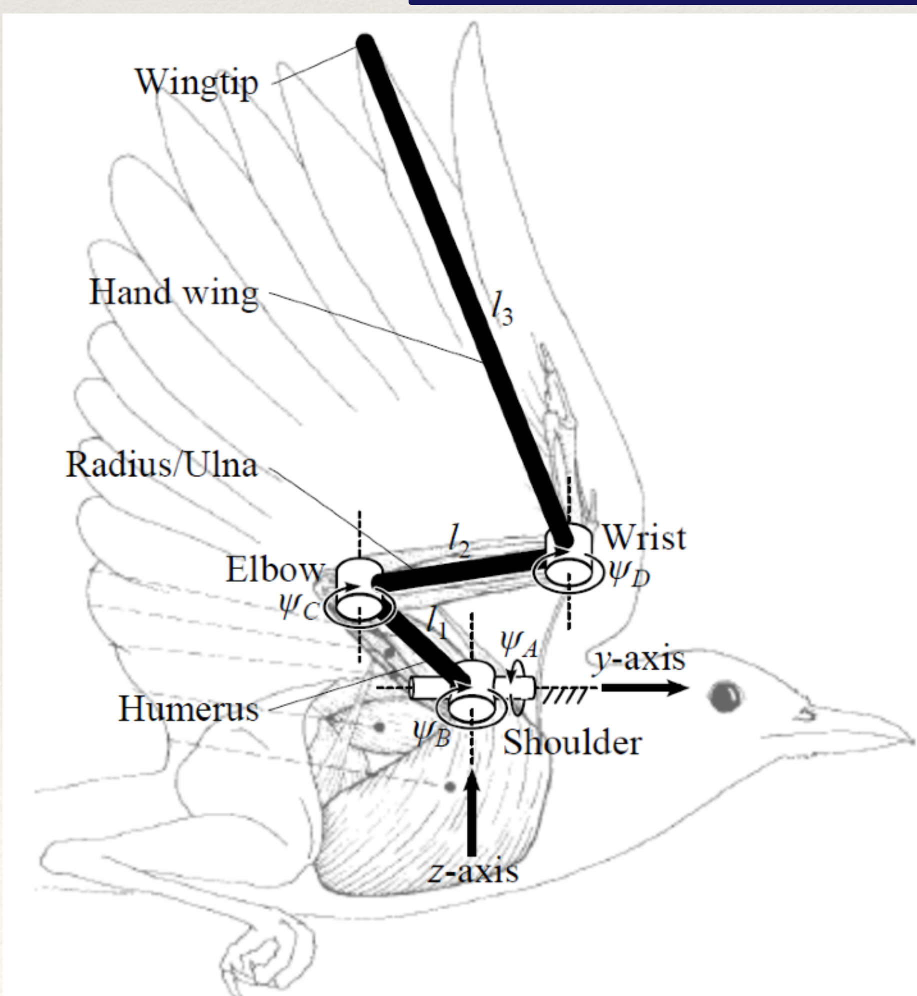
(a) Joint angle ψ_A

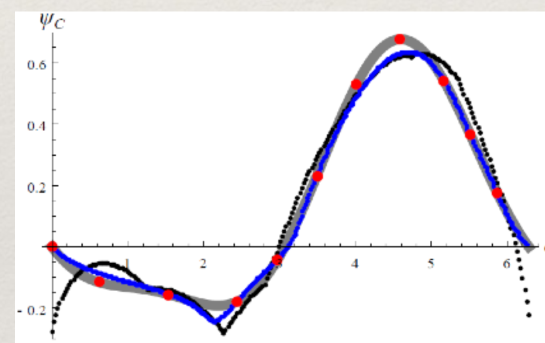




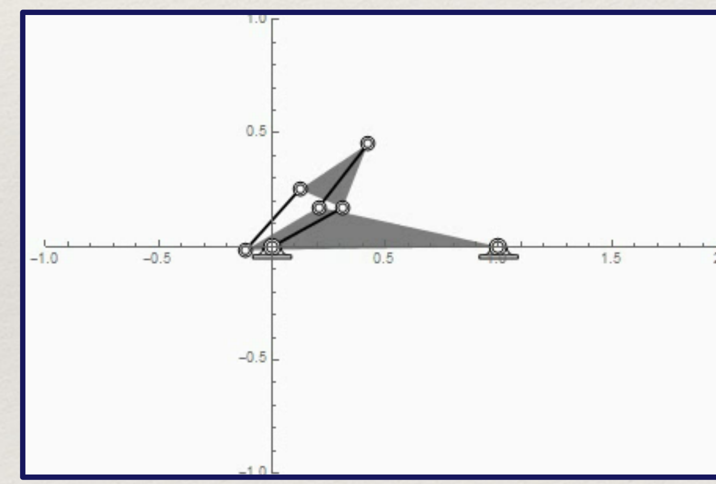
(b) Joint angle ψ_B

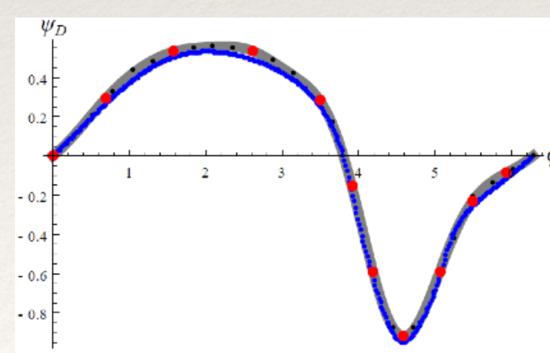




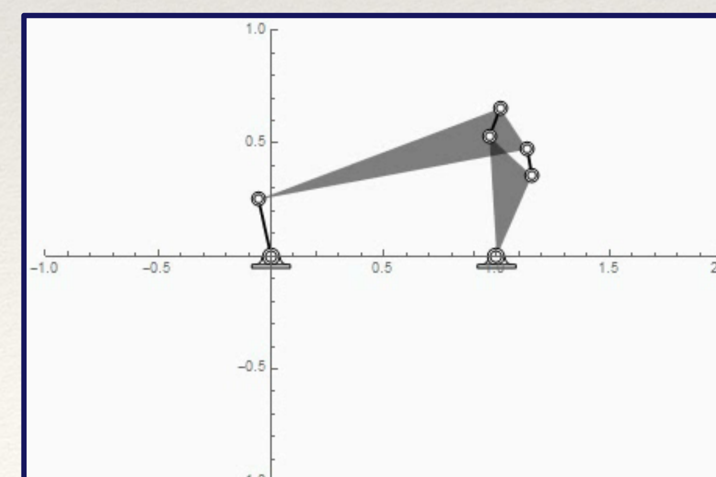


(c) Joint angle ψ_C





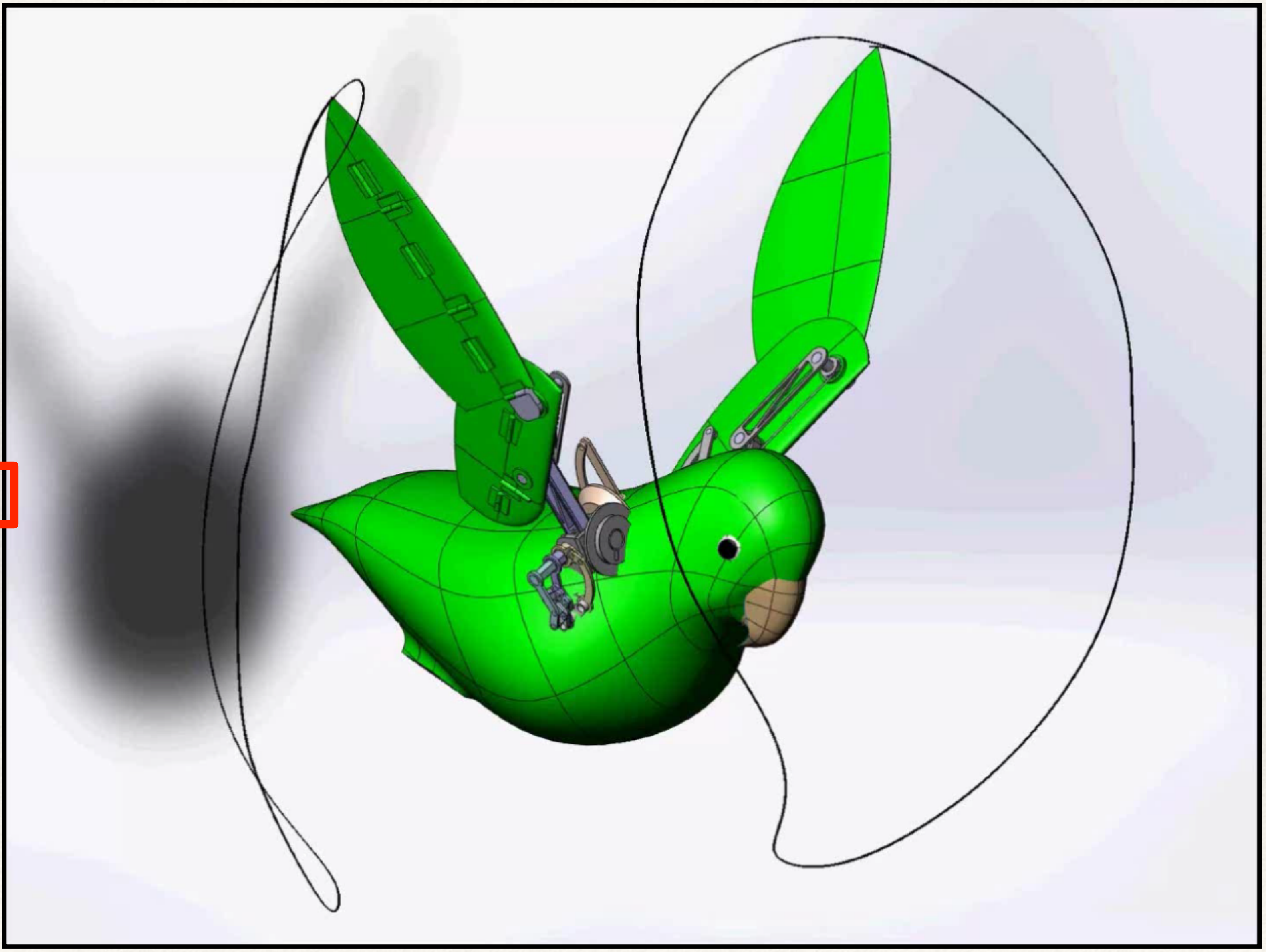
(d) Joint angle ψ_D



Biomimetic Movement

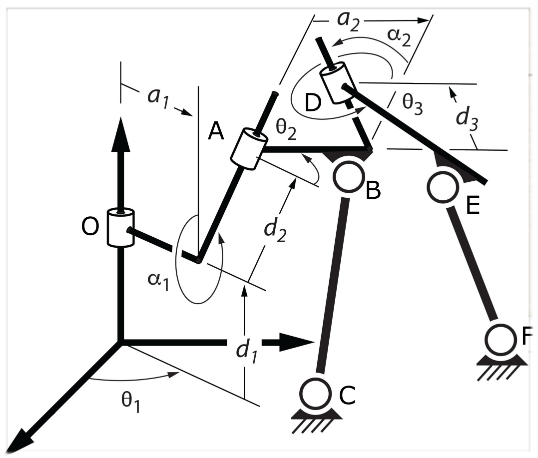
Solutions of the synthesis polynomials for each of the four target functions
$\psi_A=f_A(\phi)$, $\psi_B=f_B(\phi)$, $\psi_C=f_C(\phi)$, $\psi_D=f_D(\phi)$

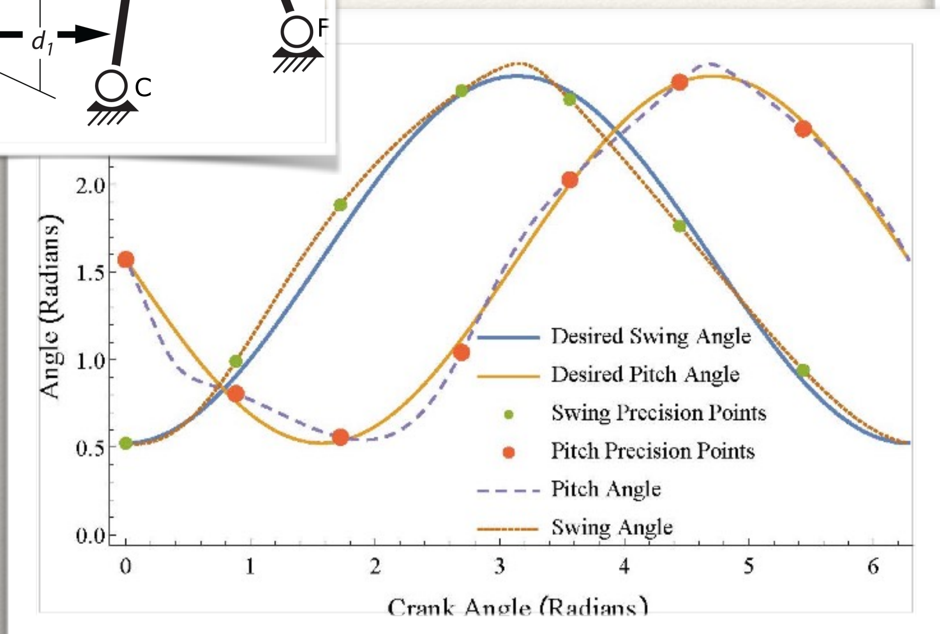
	Binary Driven			
	ψ_A	ψ_B	ψ_C	ψ_D
Linkage solutions	11428	7215	12870	11693
Design Candidates	6068	4012	7363	5775
11 point mechanisms	0	0	3	0
10 point mechanisms	0	0	12	0
9 point mechanisms	0	7	95	4
8 point mechanisms	21	54	246	95
Feasible designs	21	61	356	99
Synthesis computation time (hr)	2.2	2.0	2.5	2.2
Analysis computation time (hr)	20.2	13.4	25.6	19.1

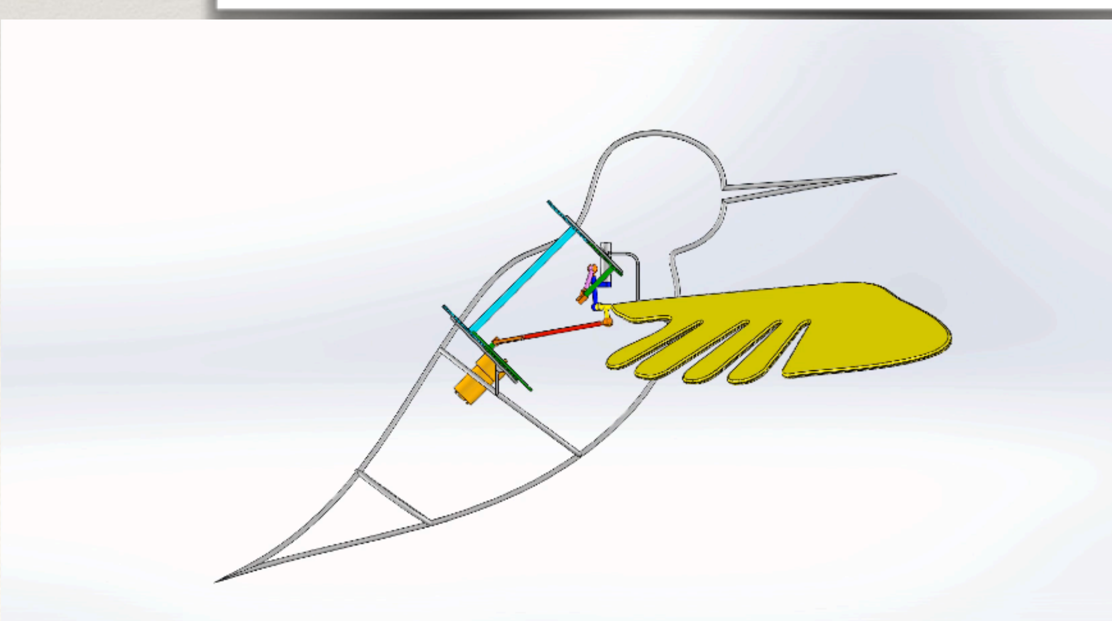


As many as $45x10^6$ possible wing linkage designs.

Hovering Wing Movement







Design and Manufacturing of Flapping Wing Mechanisms for Micro Air Vehicles

Miquel Balta[1], Khaled A. Ahmed[2], Peter L. Wang[3], J. Michael McCarthy[4] and Haithem E. Taha[5]
University of California Irvine, Irvine, CA, 92617

I. Introduction

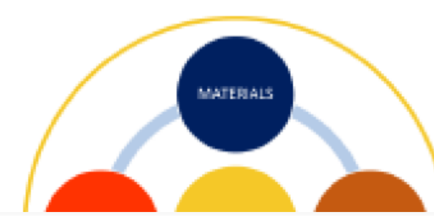
Over the last decade, Flapping Wing Micro Air Vehicles have been the aim of many of the most important researches of the scientist community. The possibility of creating a machine similar to a hummingbird or an insect has fascinated more than one generation. Thanks to the technology progressi… of passionate researchers, this dream can now become real. There have bee… universities or organizations. For example, one of the smallest FWMA… However, there is one design which has been over the rest: DARPA's FWMA…

DARPA (Defense Advance Research Project Agency) has set the defini… that the maximum dimension does not exceed 15 cm. These miniature vehic… and surveillance as well as many other applications. A special type of MAVs… birds or insects (depending on the size). Flapping insects exploit uncon… generate high lift at ultra-low Reynolds numbers. They also exploit unco… stabilize their bodies in flight and overcome gust disturbances.

In spite of the fact that DARPA has designed a FWMAV with great qua… from the University of California, Irvine has revealed that FWMAVs c… "Vibrational Controllability and Stability of FWMAV" require from a new… recently a new project about FWMAV has started at the University of Cal… undergraduate team whose aim is to create a FWMAV and apply to it this new research. Under the advice of Prof. Haithem Taha and Prof. Michael McCarthy, there are great expectations to succeed.

The goal of F.W.M.A.V. Project is to design, build, and fly a Flapping-Type Micro Air Vehicle that is capable of hovering (similar to large insects or some birds). Nevertheless, there are some differences between this FWMAV and the rest that has been done until now. These differences are shown in the main specifications of the Project. These are the following:

- Have a dimensions less than 15 cm in length, width, or height.
- Fly for more than 1 minute hovering.
- 2 degrees of freedom: Upward and Pitch.

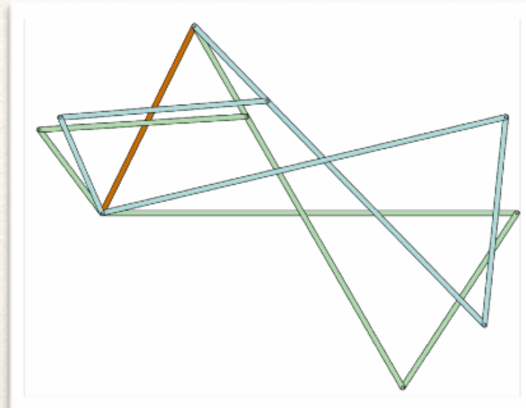


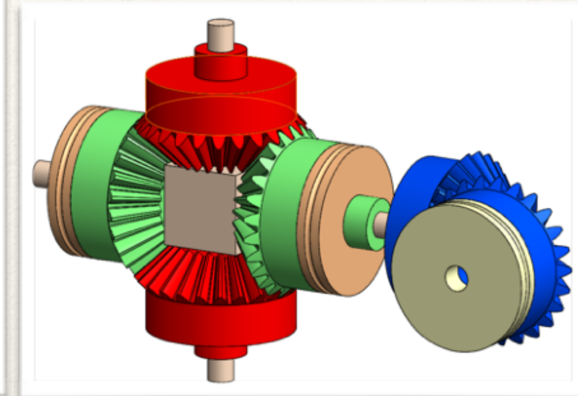
Balta,M.,Ahmed,K.A.,Wang,P.L.,McCarthy,J.M., and Taha, H. E., 2017. "Design and manufacturing of flapping wing mechanisms for micro air vehicles". In 58th AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, p. 0509.

Another Look at Mechanical Computation

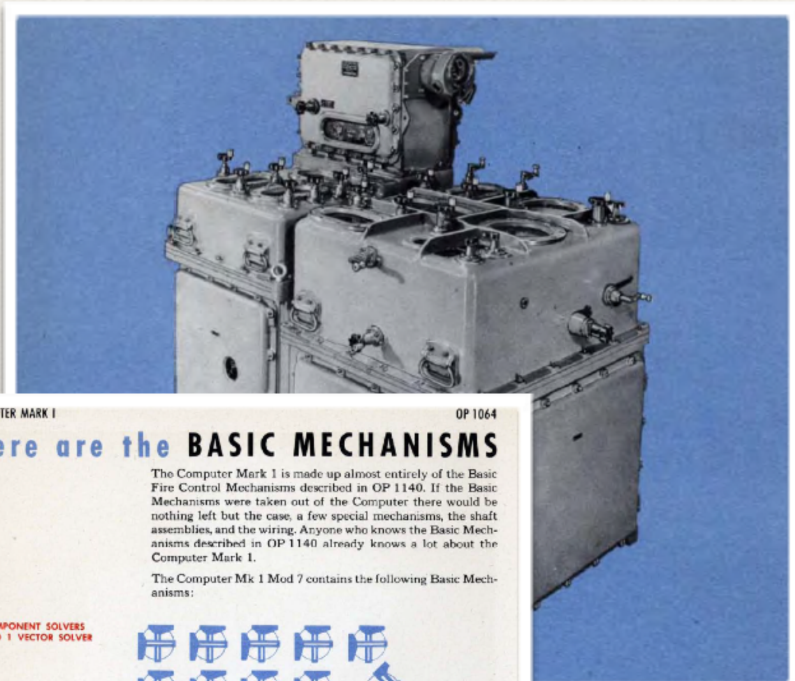
Replace Kempe's linkages for addition, negation, multiplication and translation with the equivalent elements:

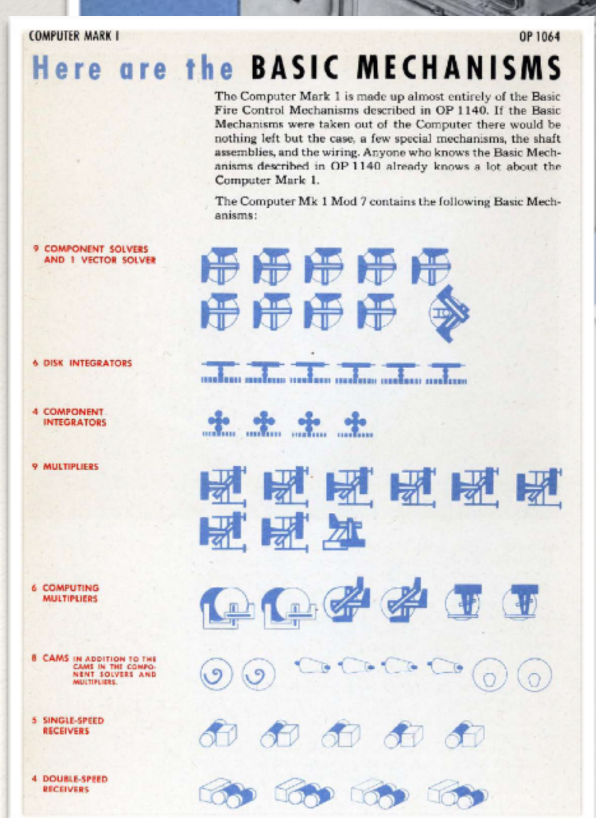
Use mechanical computation:
a bevel gear differential to add angles, and
cable drives to reverse, multiply and translate





Bevel gear differential to add.




COMPUTER MARK I OP 1064

Here are the BASIC MECHANISMS

The Computer Mark 1 is made up almost entirely of the Basic Fire Control Mechanisms described in OP 1140. If the Basic Mechanisms were taken out of the Computer there would be nothing left but the case, a few special mechanisms, the shaft assemblies, and the wiring. Anyone who knows the Basic Mechanisms described in OP 1140 already knows a lot about the Computer Mark 1.

The Computer Mk 1 Mod 7 contains the following Basic Mechanisms:

9 COMPONENT SOLVERS AND 1 VECTOR SOLVER

6 DISK INTEGRATORS

4 COMPONENT INTEGRATORS

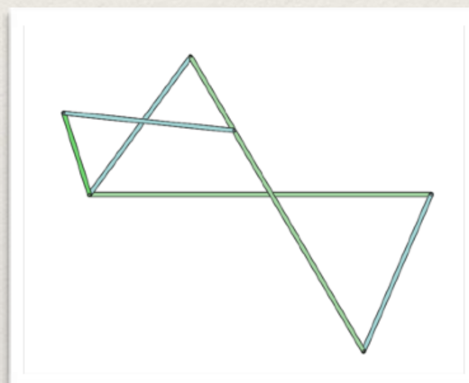
9 MULTIPLIERS

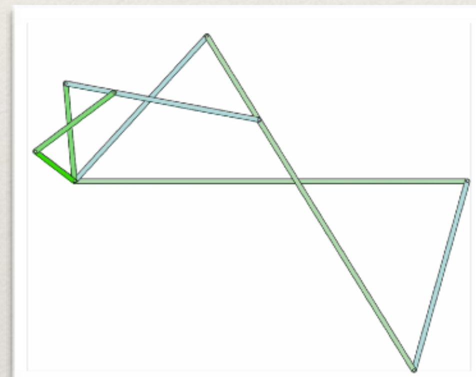
6 COMPUTING MULTIPLIERS

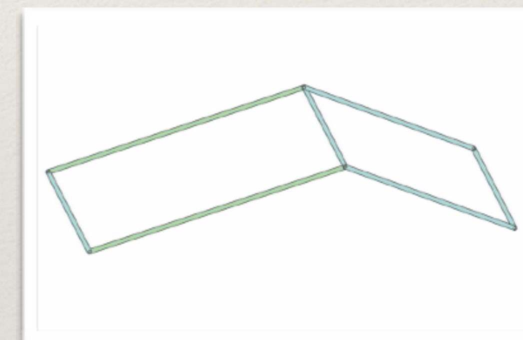
8 CAMS IN ADDITION TO THE CAMS IN THE COMPONENT SOLVERS AND MULTIPLIERS.

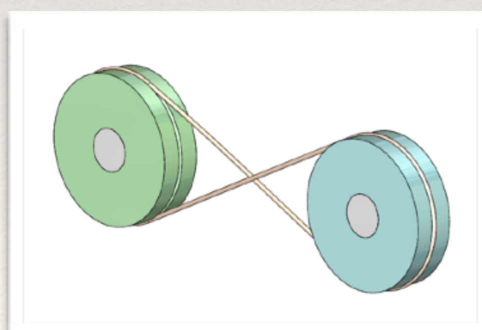
5 SINGLE-SPEED RECEIVERS

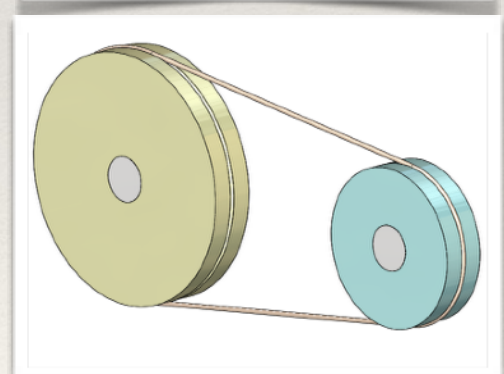
4 DOUBLE-SPEED RECEIVERS

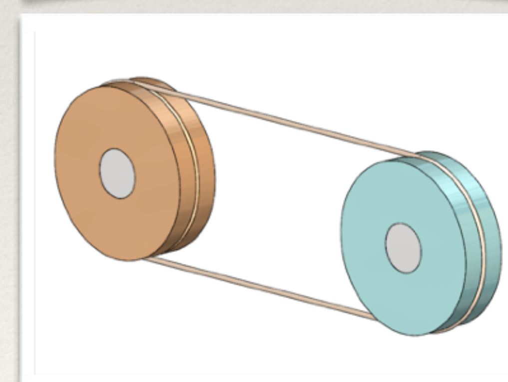












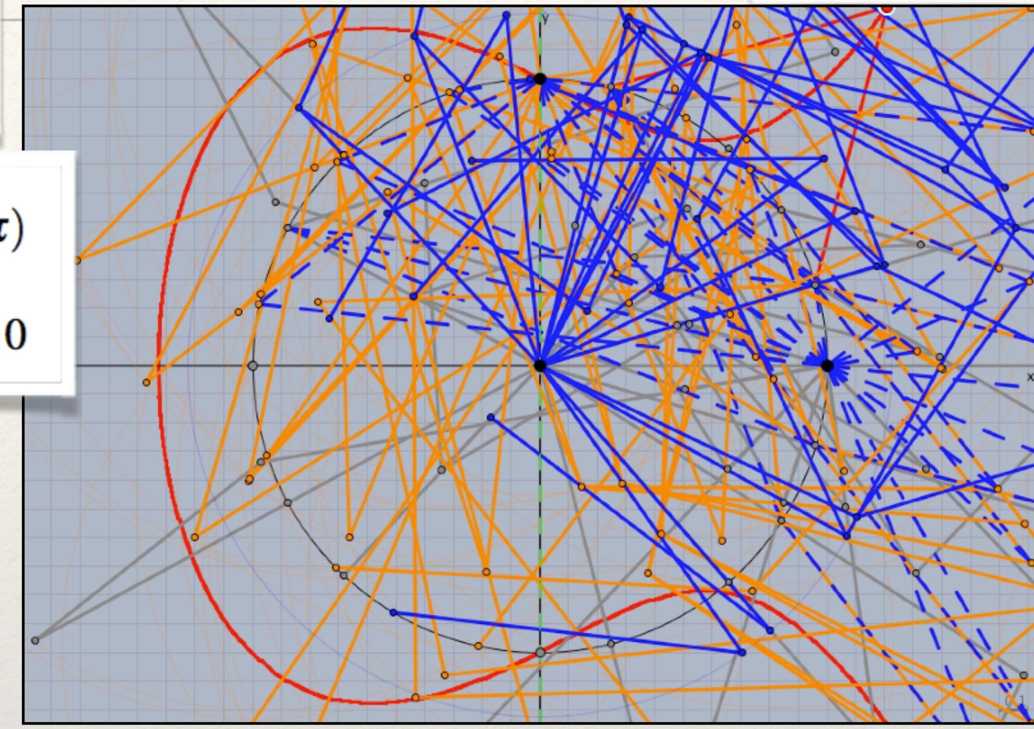
Cable drives to reverse, multiply and translate.

Drawing Algebraic Curves

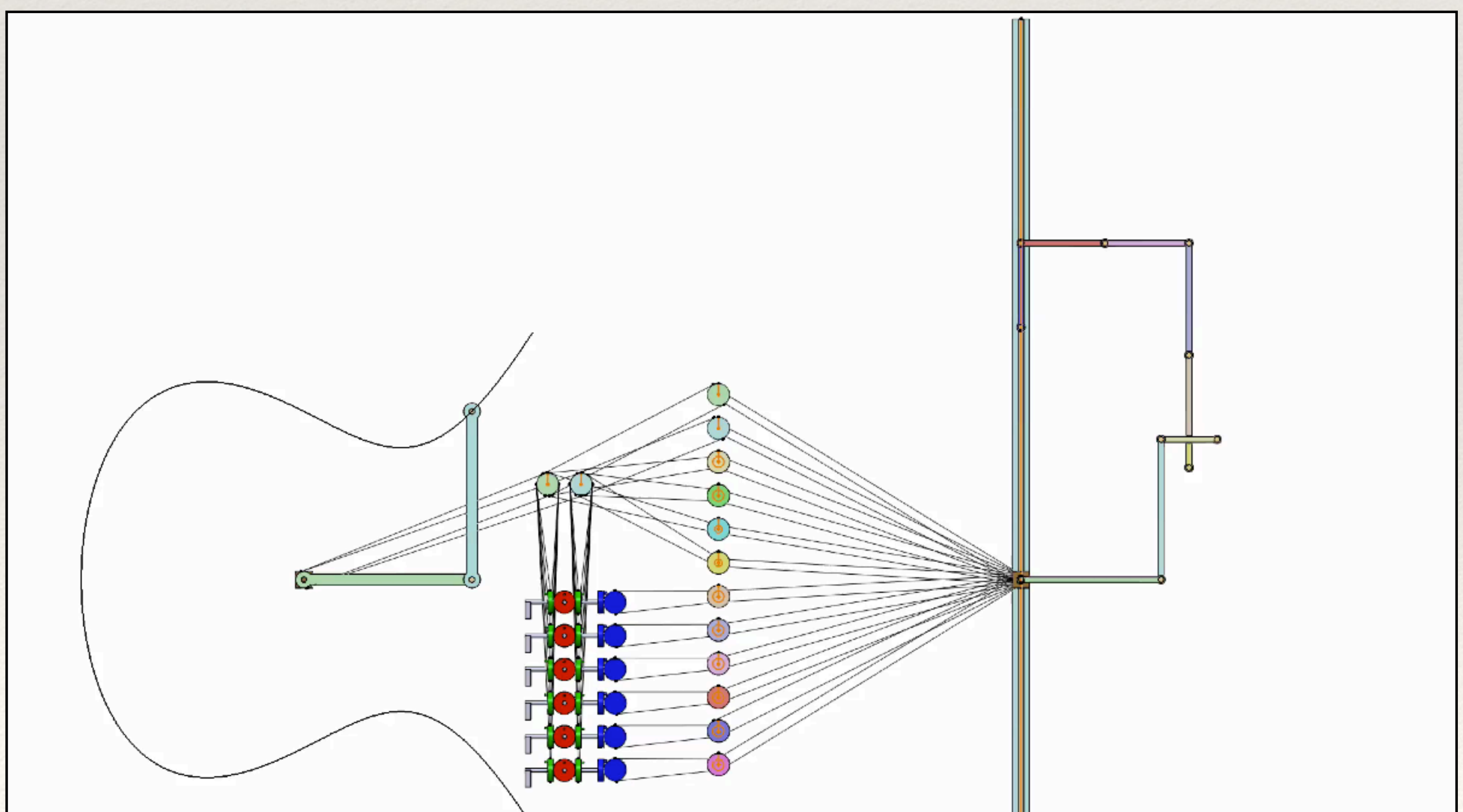
$$f(x,y) = x^3 - y^2 - x + 1 = 0$$

$$\mathbf{P} = \begin{Bmatrix} x(\theta,\phi) \\ y(\theta,\phi) \end{Bmatrix} = \begin{Bmatrix} L_1 \cos\theta + L_2 \cos\phi \\ L_1 \sin\theta + L_2 \sin\phi \end{Bmatrix}$$

$$\begin{aligned} f(\theta,\phi) = {} & \frac{5}{4}\cos\theta + \frac{5}{4}\cos\phi + \frac{1}{2}\cos 2\theta + \frac{1}{2}\cos 2\phi + \frac{1}{4}\cos 3\theta + \frac{1}{4}\cos 3\phi + \cos(\theta - \phi + \pi) \\ & + \cos(\theta + \phi) + \frac{3}{4}\cos(\theta - 2\phi) + \frac{3}{4}\cos(\theta + 2\phi) + \frac{3}{4}\cos(2\theta - \phi) + \frac{3}{4}\cos(2\theta + \phi) = 0 \end{aligned}$$



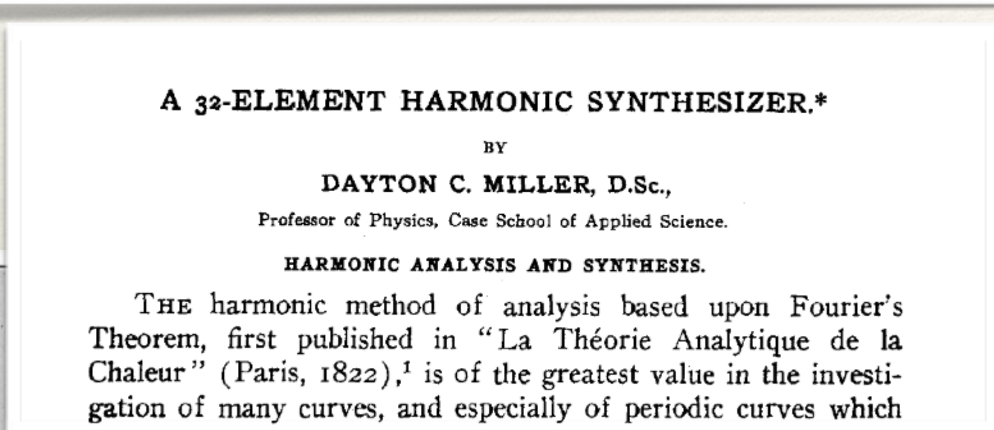
Alex Kobel (http://www.a-kobel.de/kempe/)



Link Number	Link Length	Phase	Angular Velocity
L_1	1.25	0	θ
L_2	1.25	90	ϕ
L_3	0.5	0	2θ
L_4	0.5	180	2ϕ
L_5	0.25	0	3θ
L_6	0.25	270	3ϕ
L_7	1	90	θ-ϕ
L_8	1	90	θ+ϕ
L_9	0.75	-180	θ-2ϕ
L_{10}	0.75	180	θ+2ϕ
L_{11}	0.75	-90	2θ-ϕ
L_{12}	0.75	90	2θ+ϕ

Mechanical Fourier Series

1. A closed parameterized curve, f(t)=(x(t), y(t)), has a Fourier series expansion of the coordinate functions, x(t) and y(t);
2. Scotch yoke mechanisms generate cosine and sine terms, add using a belt and pulleys;
3. Combine the movements of x and y coordinates to draw the curve.



A 32-ELEMENT HARMONIC SYNTHESIZER.*

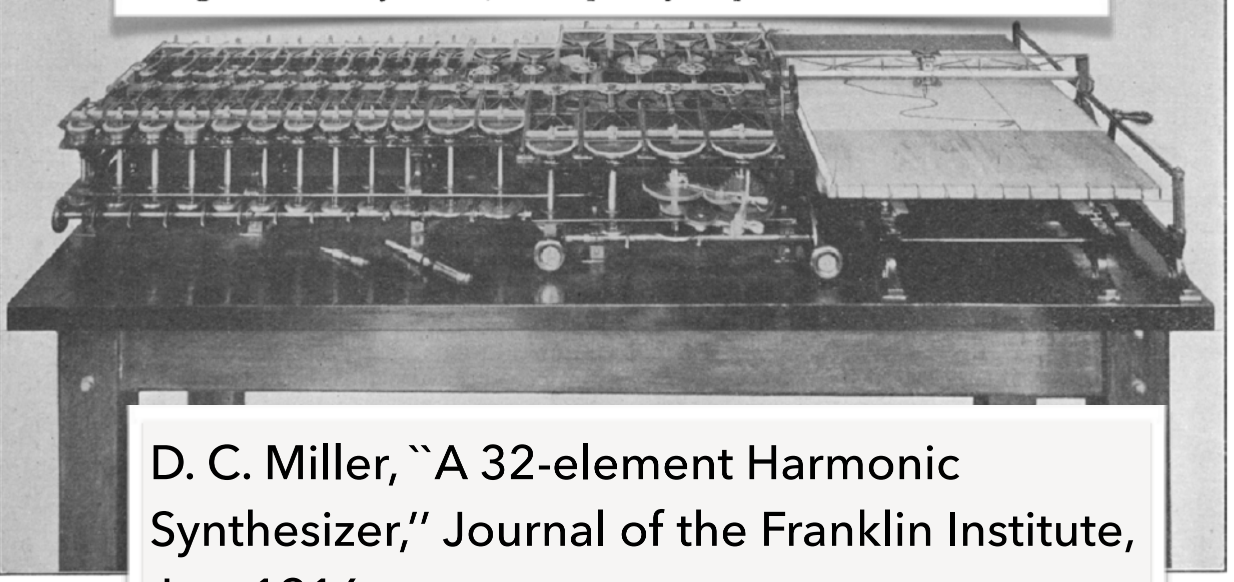
BY

DAYTON C. MILLER, D.Sc.,

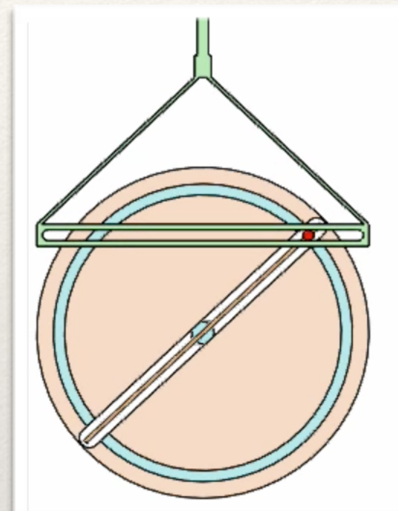
Professor of Physics, Case School of Applied Science.

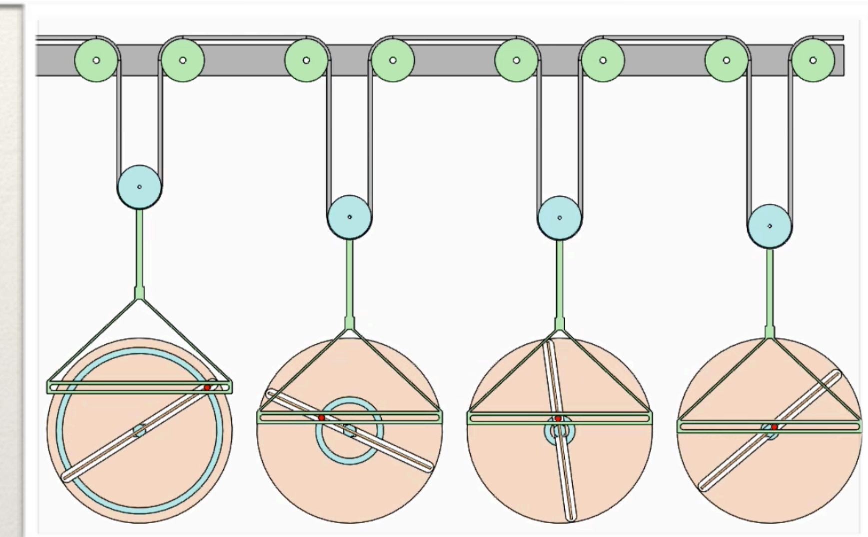
HARMONIC ANALYSIS AND SYNTHESIS.

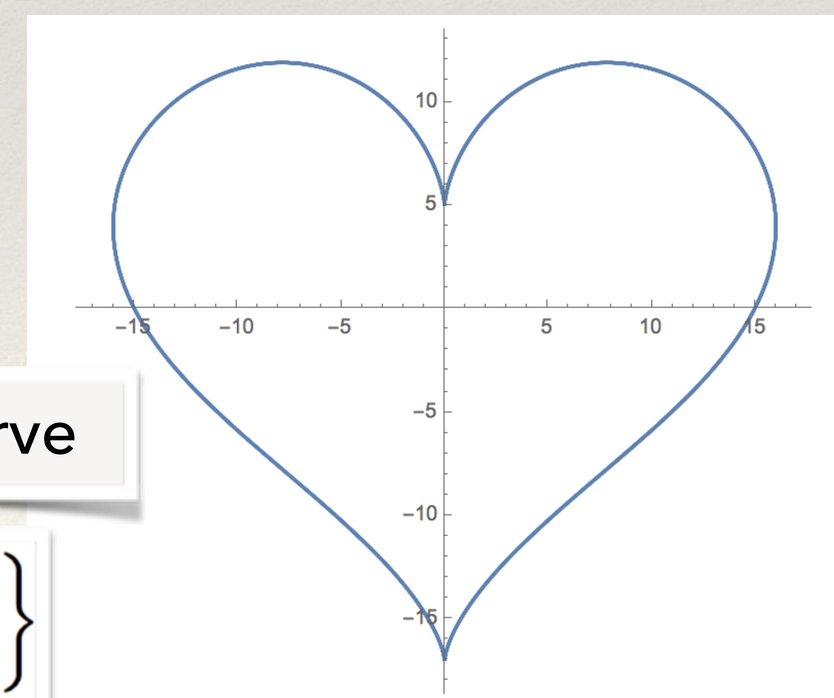
THE harmonic method of analysis based upon Fourier's Theorem, first published in "La Théorie Analytique de la Chaleur" (Paris, 1822),[1] is of the greatest value in the investigation of many curves, and especially of periodic curves which



D. C. Miller, ``A 32-element Harmonic Synthesizer,'' Journal of the Franklin Institute, Jan. 1916.





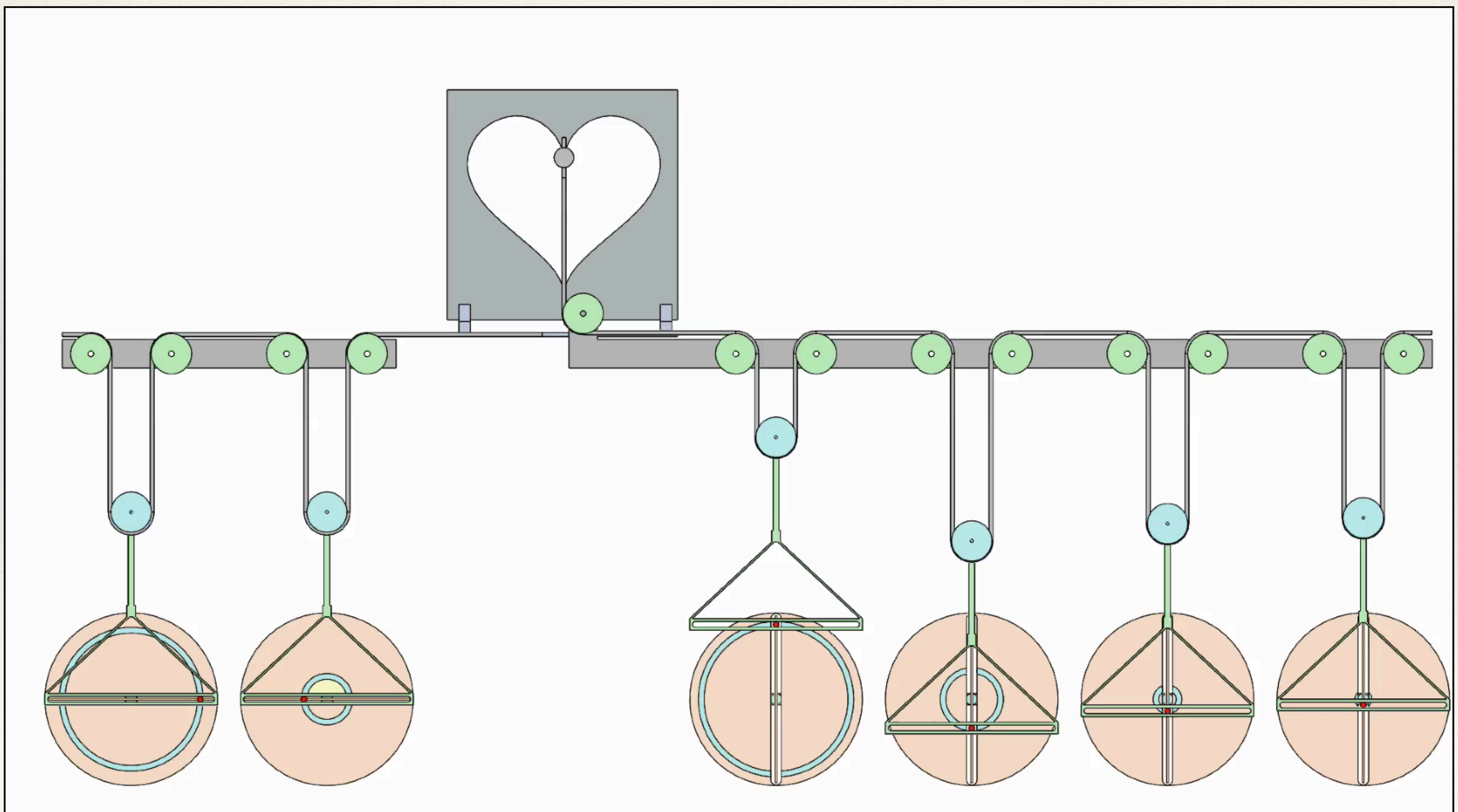


The Heart Curve

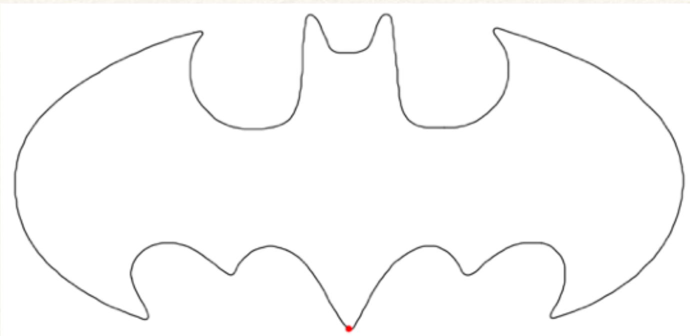
$$\begin{Bmatrix} x(t) \\ y(t) \end{Bmatrix} = \begin{Bmatrix} 12\sin t + 4\sin(-\pi + 3t) \\ 13\sin(\frac{\pi}{2} + t) + 5\sin(\frac{3\pi}{2} + 2t) + 2\sin(\frac{3\pi}{2} + 3t) + \sin(\frac{3\pi}{2} + 4t) \end{Bmatrix}$$

Trigonometric Plane Curves

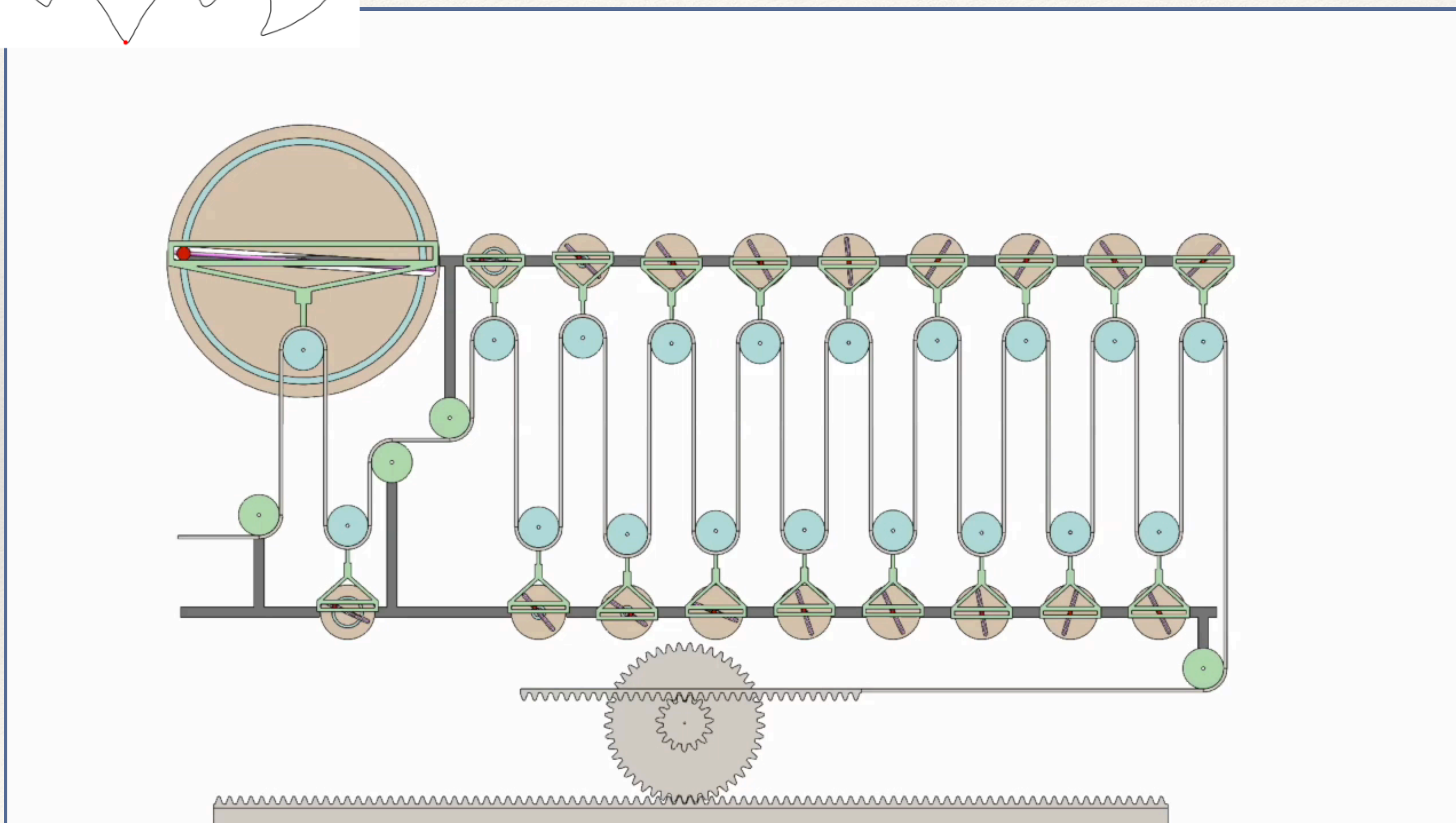
$$\begin{Bmatrix} x(t) \\ y(t) \end{Bmatrix} = \begin{Bmatrix} 12\sin t + 4\sin(-\pi + 3t) \\ 13\sin(\frac{\pi}{2} + t) + 5\sin(\frac{3\pi}{2} + 2t) + 2\sin(\frac{3\pi}{2} + 3t) + \sin(\frac{3\pi}{2} + 4t) \end{Bmatrix} = \begin{Bmatrix} \sum_{k=0}^{m} a_k \cos k\theta + b_k \sin k\theta \\ \sum_{k=0}^{m} c_k \cos k\theta + d_k \sin k\theta \end{Bmatrix} \quad (1)$$



Curves Defined by Points

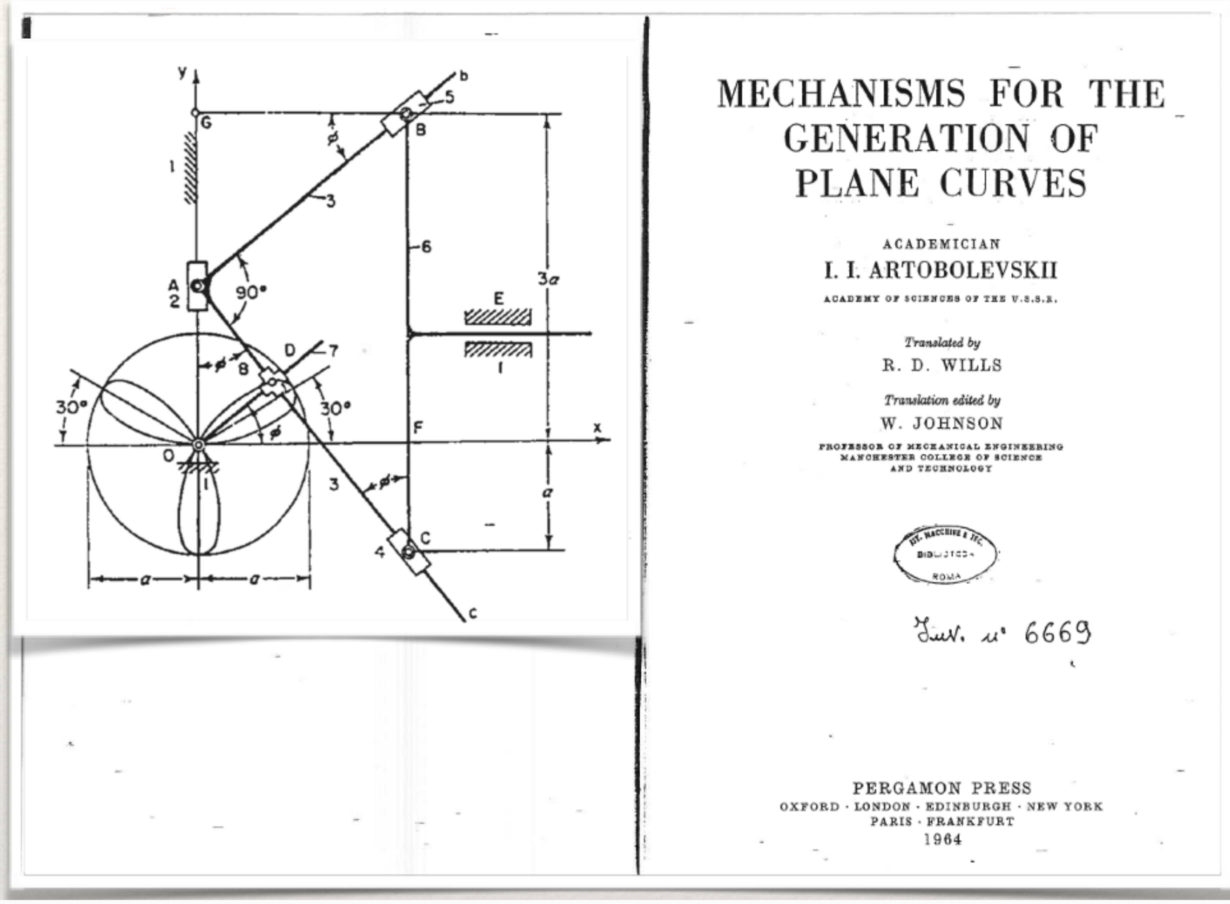


The Discrete Fourier Transform of boundary points yields a trigonometric curve z(t)=(x(t), y(t)).



The logo boundary of 3235 points yielded 19 terms for x(t) and y(t).

Epicycles: The Trifolium Curve



MECHANISMS FOR THE GENERATION OF PLANE CURVES

ACADEMICIAN

I. I. ARTOBOLEVSKII

ACADEMY OF SCIENCES OF THE U.S.S.R.

Translated by

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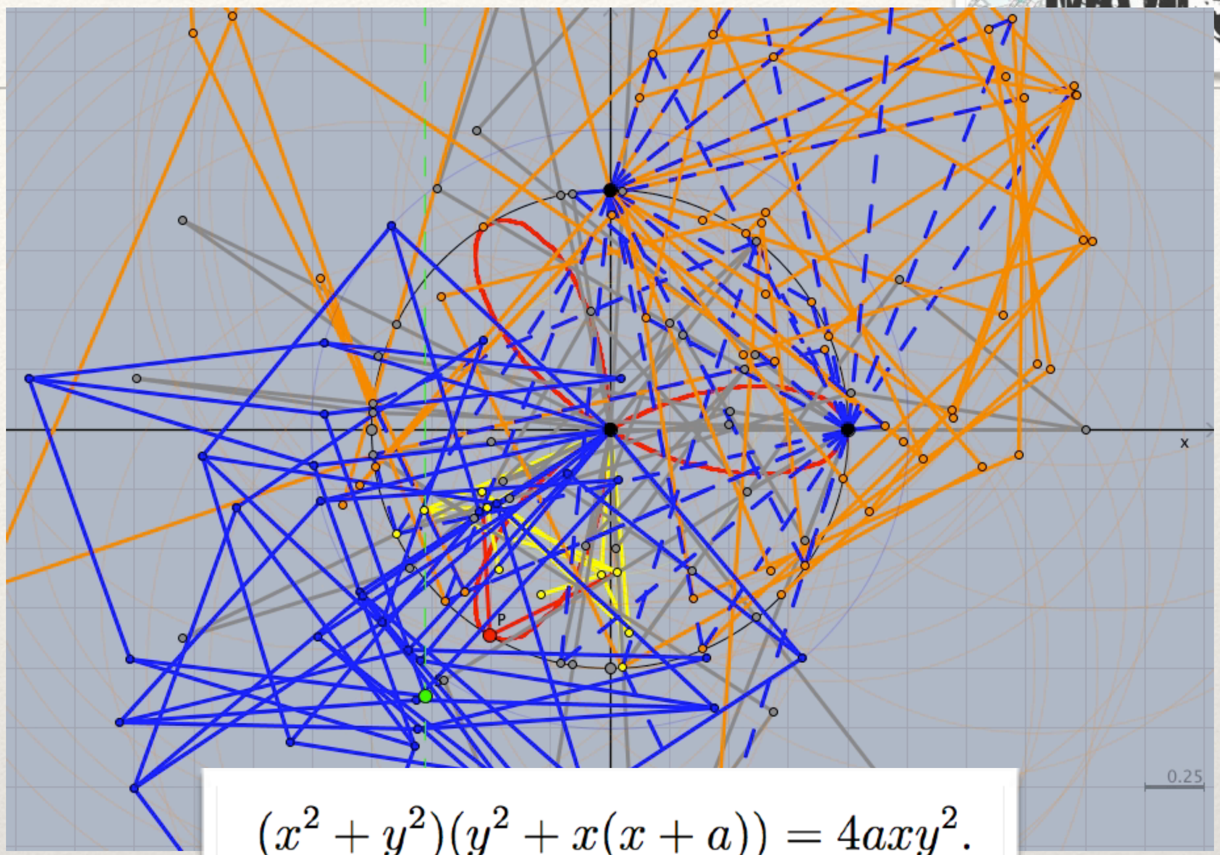
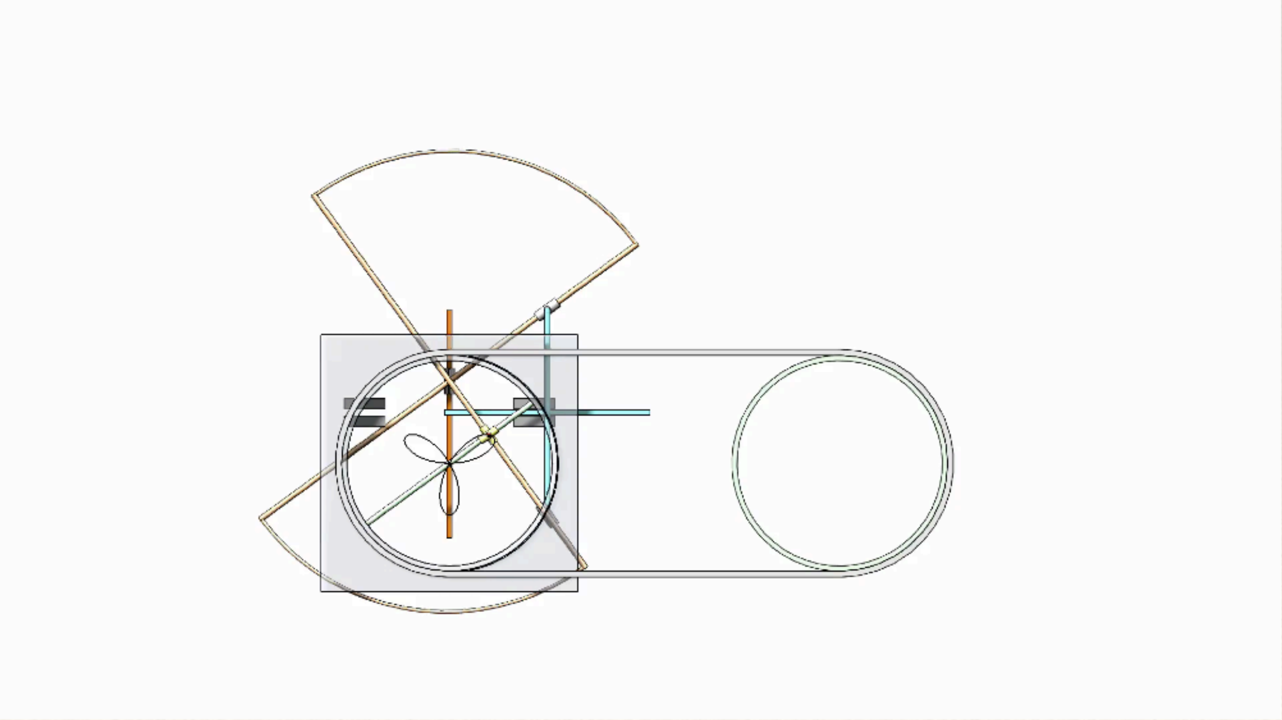
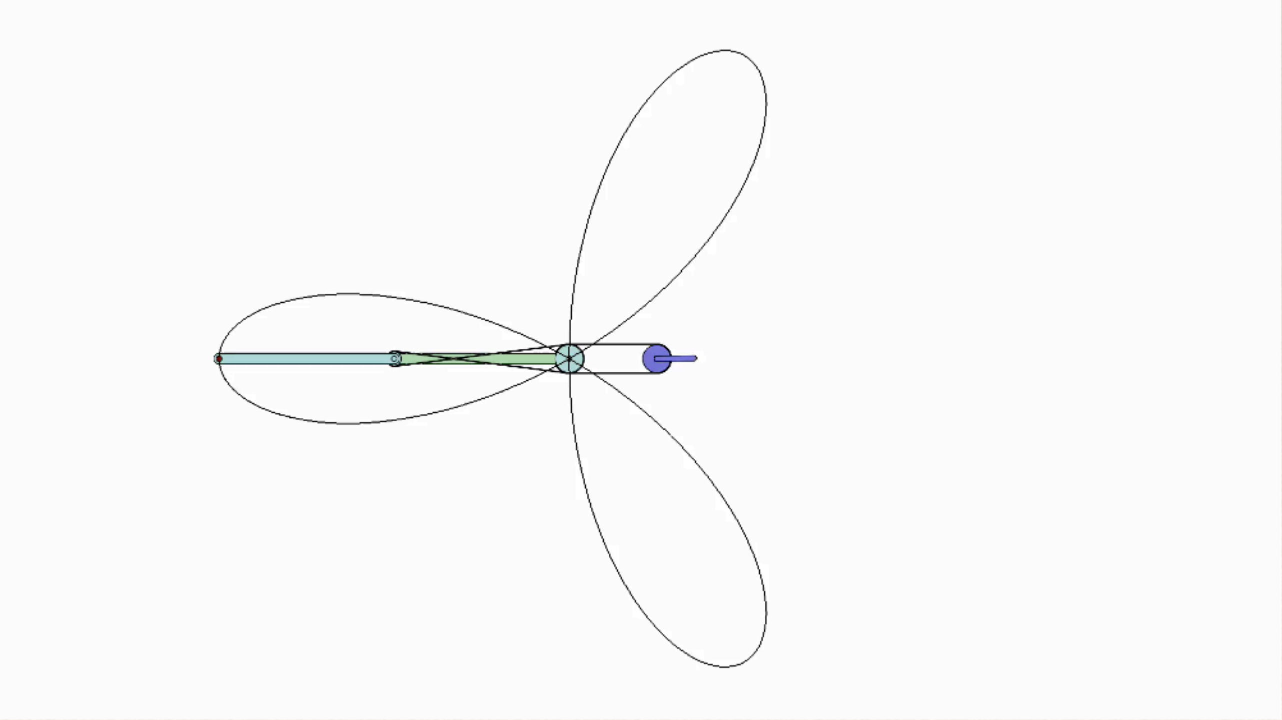


$$(x^2+y^2)(y^2+x(x+a)) = 4axy^2.$$

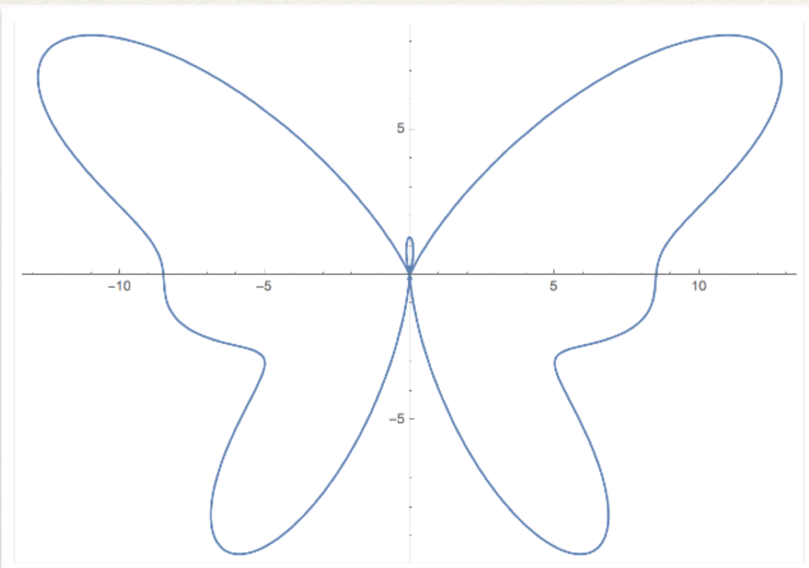


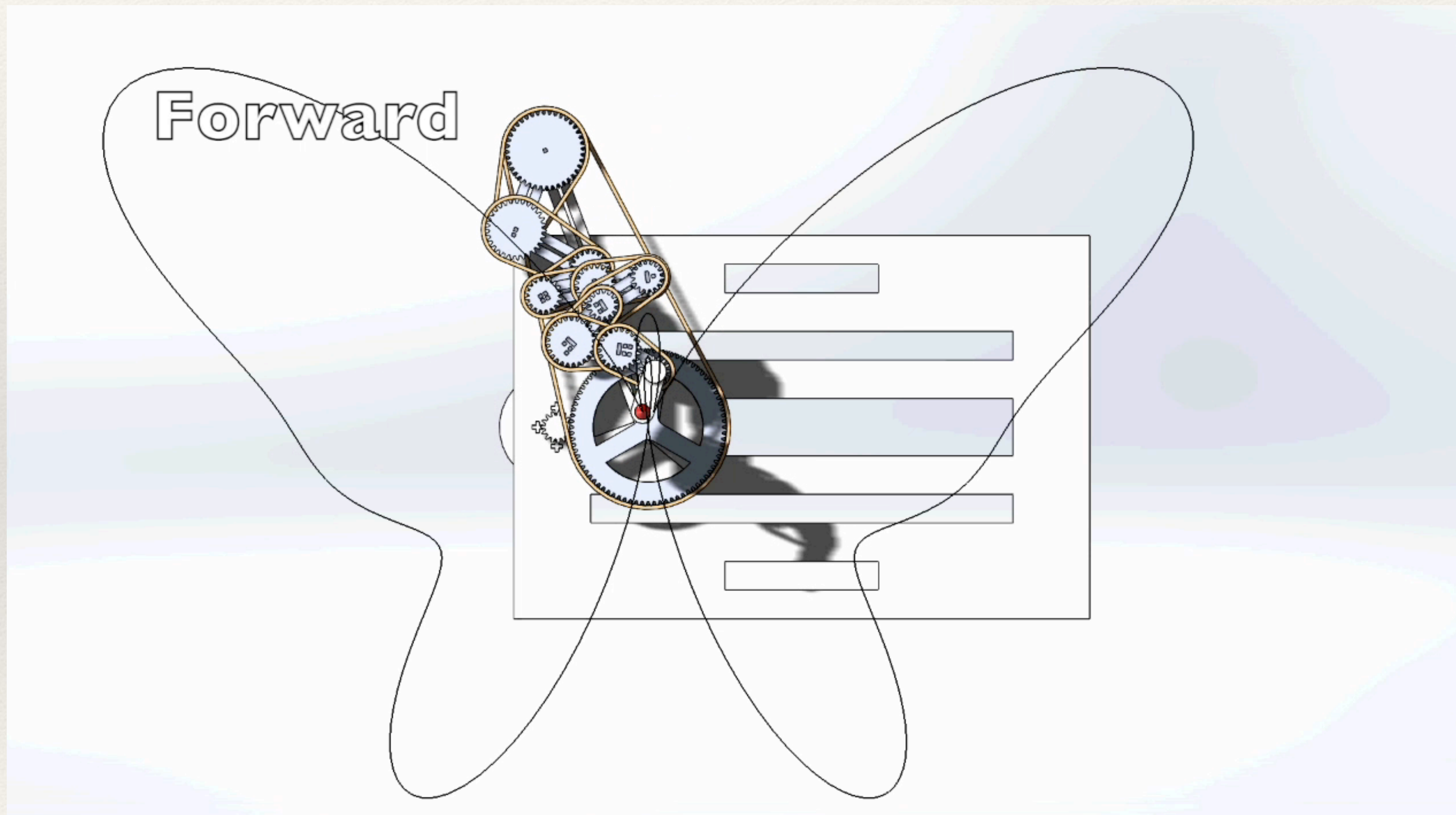


Epicycles: The Coupled Serial Chain

The Butterfly mechanism draws the Butterfly curve:

$$\mathbf{z}(t) = \begin{Bmatrix} x(t) \\ y(t) \end{Bmatrix} = \begin{cases} 9\cos(t) + 0.75\cos(3t) - 1.25\cos(5t) + 0.65\sin(2t) + 2.4\sin(4t) + 0.25\sin(6t) - 1.2\sin(8t) - 0.2\sin(10t) \\ -0.5 + 1.65\cos(2t) + 0.1\cos(4t) - 2.25\cos(6t) + 0.8\cos(8t) + 0.2\cos(10t) + 5\sin(t) + 3.25\sin(3t) - 1.25\sin(5t) \end{cases}$$





Y. Liu and J. M. McCarthy, "Design of Mechanisms to Draw Trigonometric Plane Curves," special issue “Selected Papers from the IDETC 2016,” Journal of Mechanisms and Robotics, April 2017, Vol 9(2). doi: 10.1115/1.4035882

The Trigonometric Bezier Curve

A cubic Bezier curve is defined by four control points.

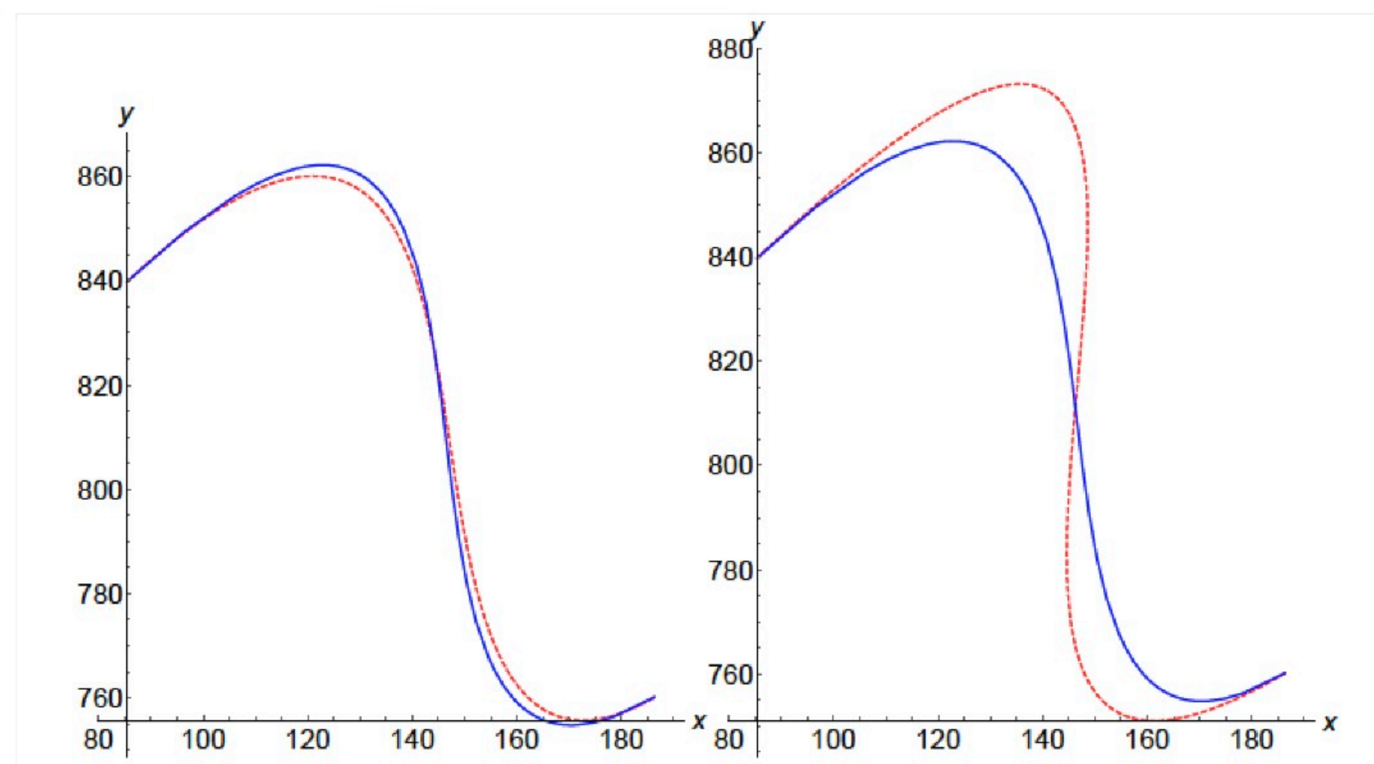
$$\mathbf{r}(t) = (1-t)^3\mathbf{P}_0 + 3(1-t)^2t\mathbf{P}_1 + 3(1-t)t^2\mathbf{P}_2 + t^3\mathbf{P}_3, \quad 0 \leqslant t \leqslant 1.$$

A cubic Trigonometric Bezier curve can be defined by the same four control points.

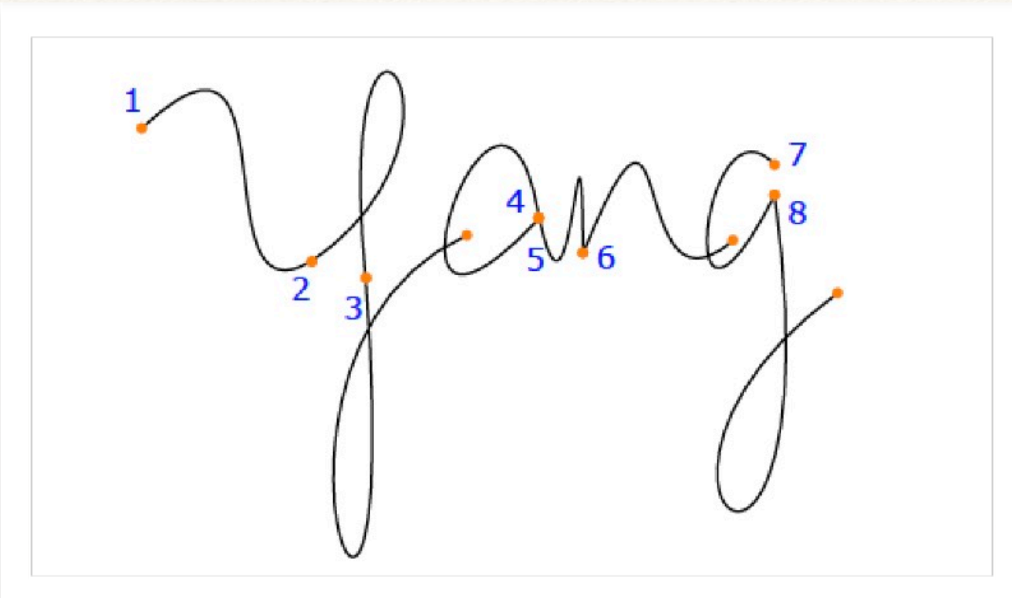
$$\begin{aligned} \mathrm{s}(t,\lambda) &= (1-\sin\tfrac{\pi t}{2})^2(1-\lambda\sin\tfrac{\pi t}{2})\mathbf{P}_0 + \sin\tfrac{\pi t}{2}(1-\sin\tfrac{\pi t}{2})(2+\lambda(1-\sin\tfrac{\pi t}{2}))\mathbf{P}_1 \\ &+ \cos\tfrac{\pi t}{2}(1-\cos\tfrac{\pi t}{2})(2+\lambda(1-\cos\tfrac{\pi t}{2}))\mathbf{P}_2 + (1-\cos\tfrac{\pi t}{2})^2(1-\lambda\cos\tfrac{\pi t}{2})\mathbf{P}_3, \\ & \qquad 0 \leqslant t \leqslant 1. \end{aligned}$$

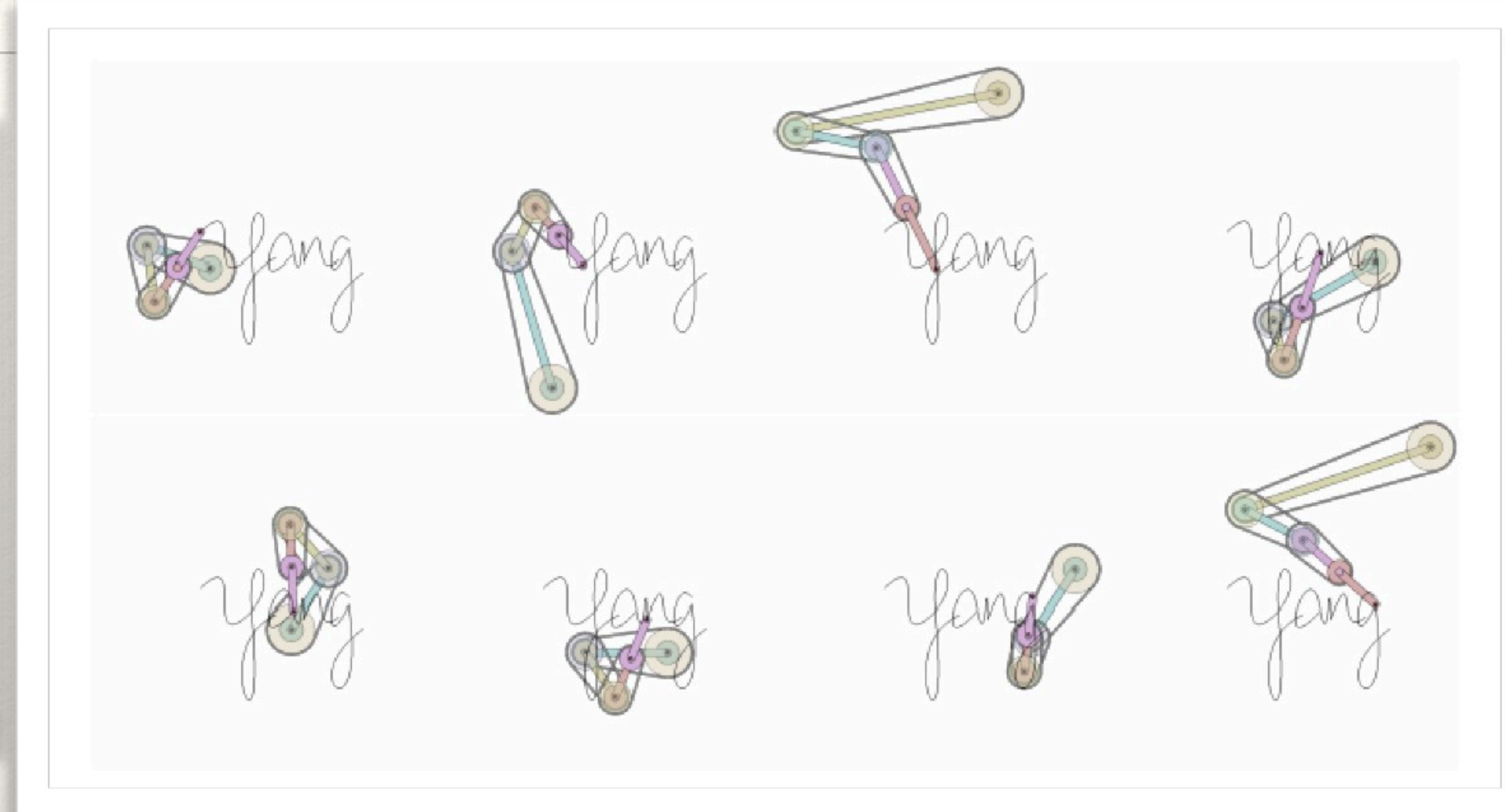
The shape parameter provides adjustment of the shape of the trigonometric Bezier curve to fit the original Bezier curve.

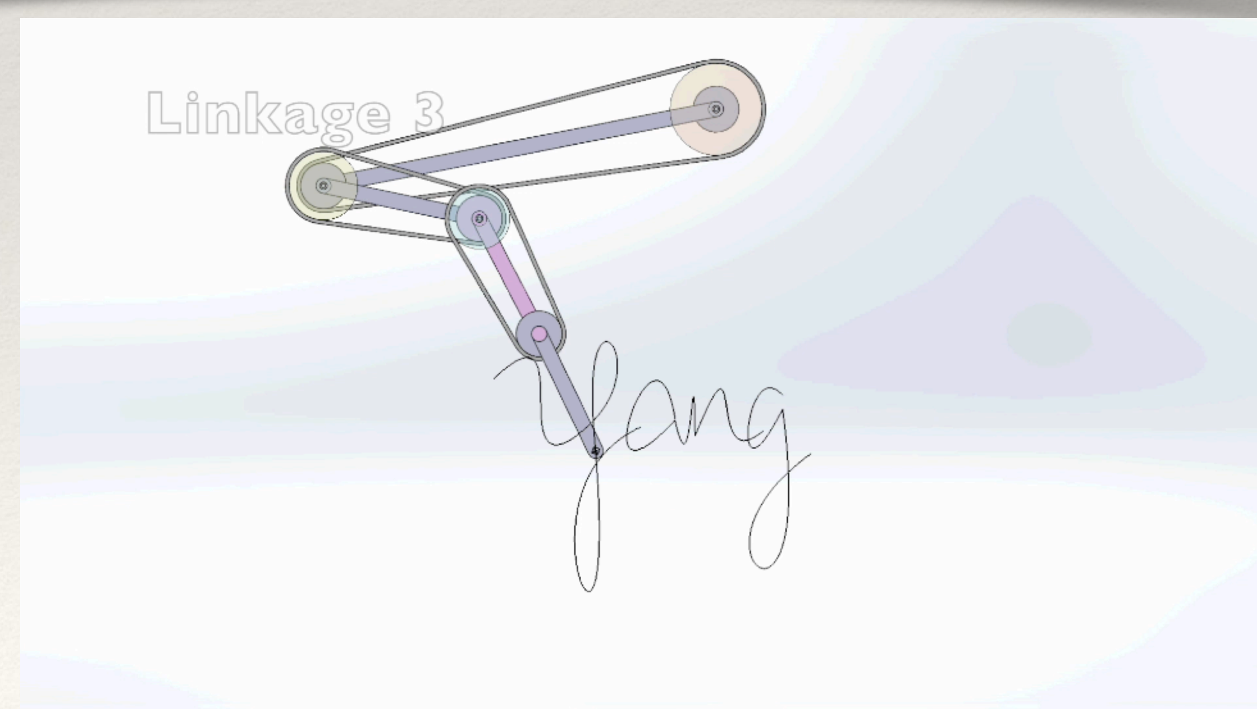
Y. Liu and J. M. McCarthy, "Design of a Linkage to Draw a Bezier Curve." Submitted to Mechanism and Machine Theory.



A Linkage Signs Your Name



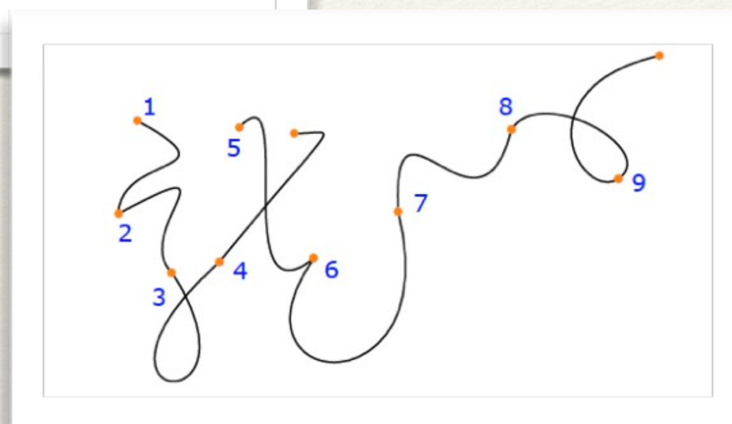


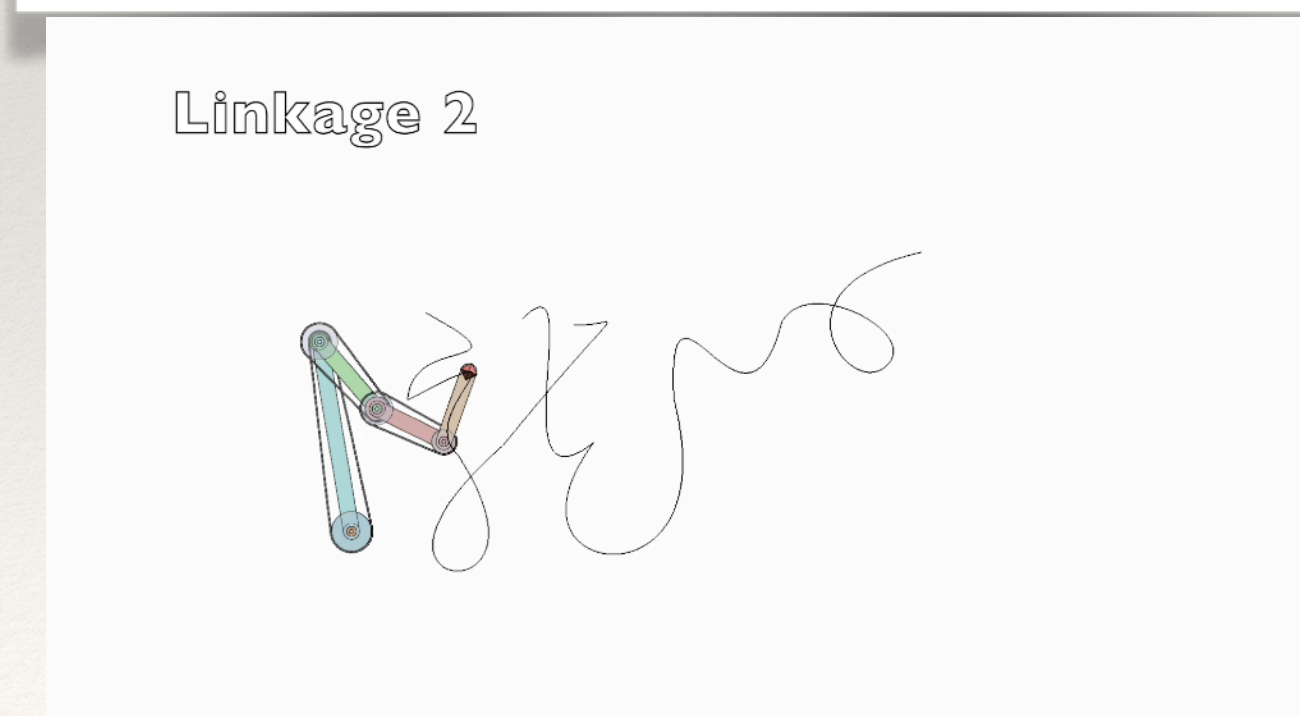


Each of the serial chains is driven by the same input.

The system has one degree-of-freedom

A Linkage Writes Chinese





Each of the serial chains is driven by the same input.

The system has one degree-of-freedom

Inventors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UCI ENERGY INVITATIONAL
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DESIGN REVIEW
APRIL 21, 2017, 9AM - 1PM
OC FAIR & EVENT CENTER
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COMPETITION DAY
MAY 6, 2017, 8AM - 3PM
UCI LOT 16
HOW FAST CAN YOU GO ON
ONE DOLLAR'S WORTH OF ENERGY?
Contact Prof. McCarthy at jmmccart@uci.edu

Conclusions

- *Path synthesis of a four-bar coupler curve through nine points has been solved. Path synthesis of a six-bar coupler curve through 15 points is unsolved, because is beyond our computational capabilities.*

- *Kempe's Universality Theorem, proves the existence of a drawing linkage for every algebraic curve, but the construction yields complex systems with hundreds of links for a cubic curve.*

- *Fourier approximation yields trigonometric plane curves that can be drawn by a coupled serial chain.*

- *Trigonometric cubic Bezier curves can be drawn by four-link coupled serial chains. Thus, a one degree-of-freedom linkage system can sign your name and write cursive Chinese.*

- *Kinematic synthesis of robotic systems to draw curves contributes to design innovation. Opportunities are as varied as stroke rehabilitation, disaster relief, vehicle suspensions, and walking and flying robots.*

Thank you, do you have any questions?