Algebraic Geometry for Projection Kinematic Analysis of DNA Origami Nano-Mechanisms

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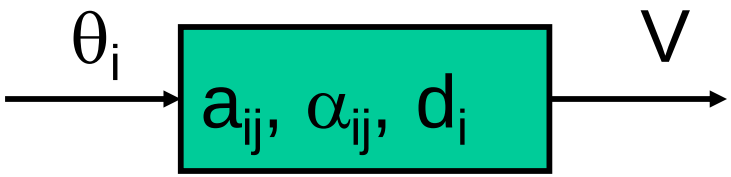
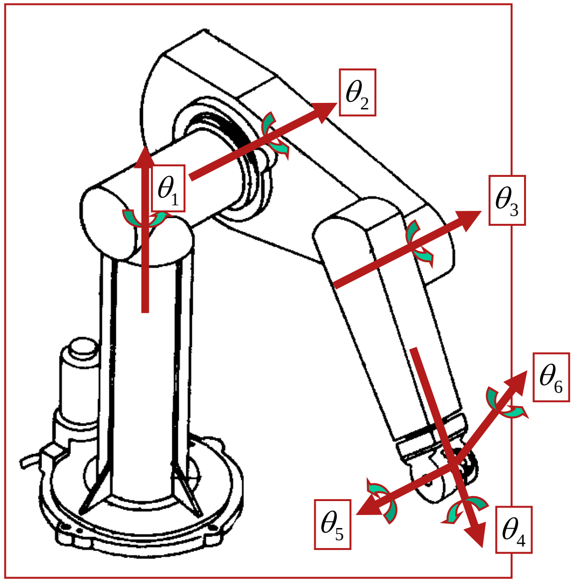
Outline

- Kinematic analysis and synthesis of rigid body mechanisms using polynomial homotopy
- Kinetostatic analysis and synthesis of compliant mechanisms using polynomial homotopy
- Design of DNA origami mechanisms (DOM)
- Configuration Analysis of DNA Origami Mechanisms via Projection Kinematics

Kinematic Analysis and Synthesis of Rigid Body and Compliant Mechanisms Using Polynomial Homotopy

Kinematics Problems





Problems	Dimensions	Joint Actuations	End-effector
Forward Analysis	✓given	✓given	? unknown
Inverse Analysis	✓given	? unknown	✓given
Dimensional Synthesis	? unknown	? unknown	✓given

- **They are all about for solving polynomial systems**
  - Multiple solutions

Milestone Problems in Kinematics

16

- **Inverse Kinematics of 6R Manipulator (1980s)**
  - Tsai, L.W., Morgan, A. (1985), 1-homogeneous homotopy, 256 paths
  - Morgan and Sommes (1987), 2-homogeneous homotopy, 96 paths
  - Wampler and Morgan (1989), parameter homotopy, 16 paths

40

- **Forward Kinematics of 6-6 Stewart-Gouph Parallel Platform Manipulator (1990s)**
  - Raghavan, M and Roth, B., (1993), 960 paths
  - Wampler (1996), 84 paths

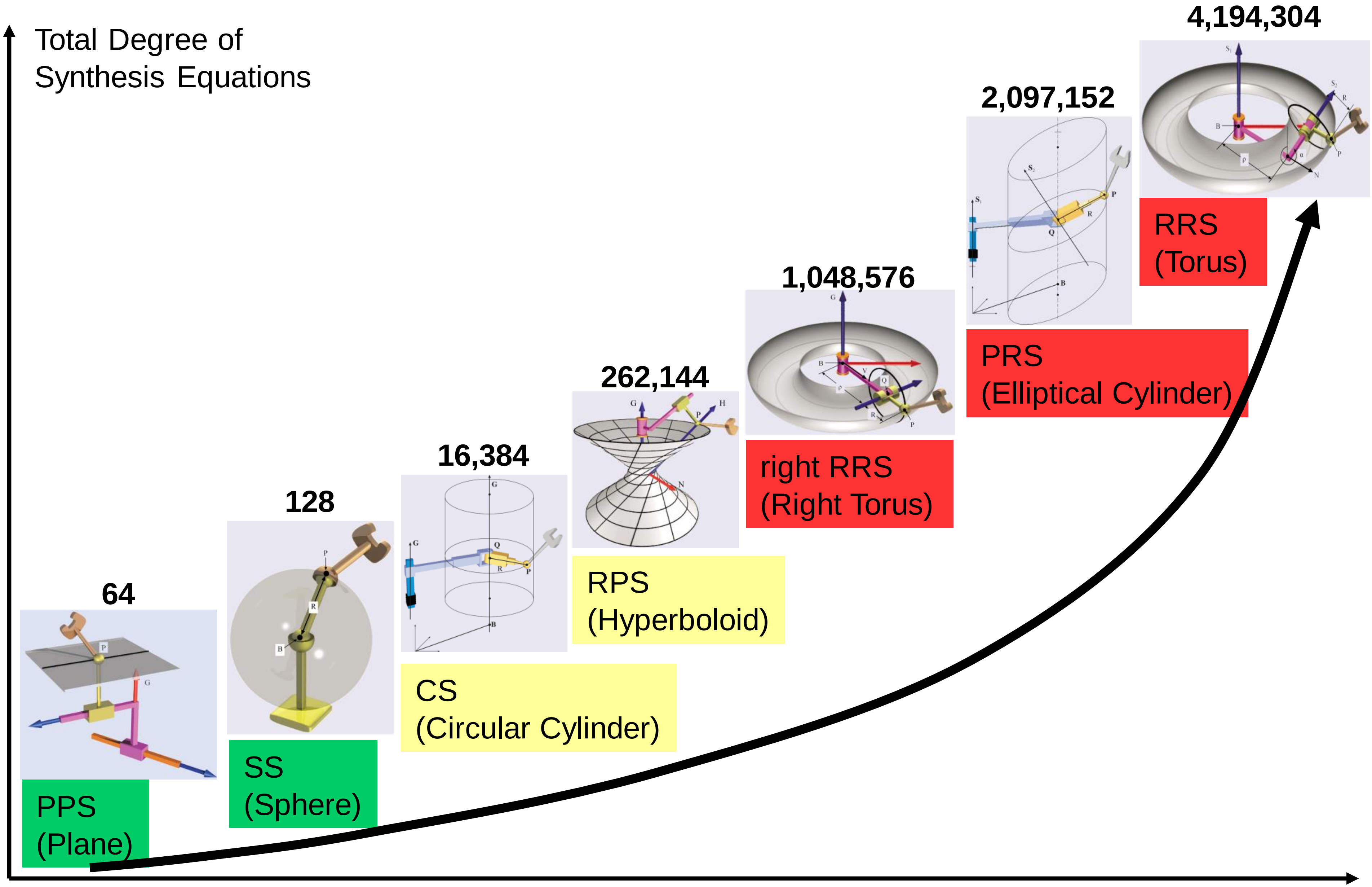
1442

- **Nine point Synthesis of Planar-4 bar (1990s)**
  - Wampler, Morgan and Sommese (1992), 143,360 paths
- **Spatial Rigid Body Guidance Problems (2000s)**
- **Kinetostatics of compliant mechanisms (2000s)**

Complexity

- **Six-bar function/path generation problems (2010s)**

Complexity of Mechanism Synthesis Problems

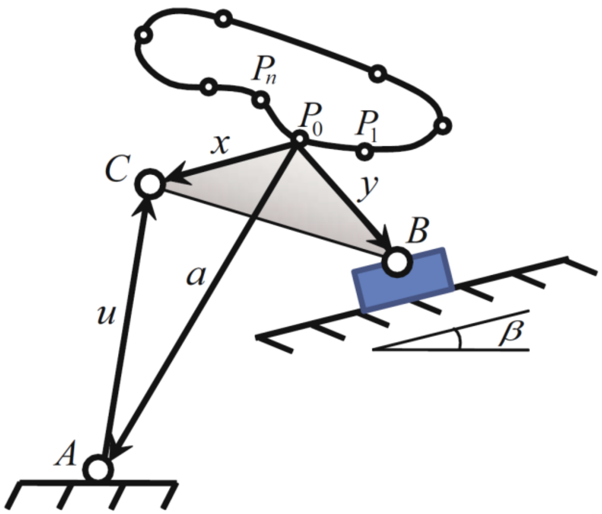


Solutions

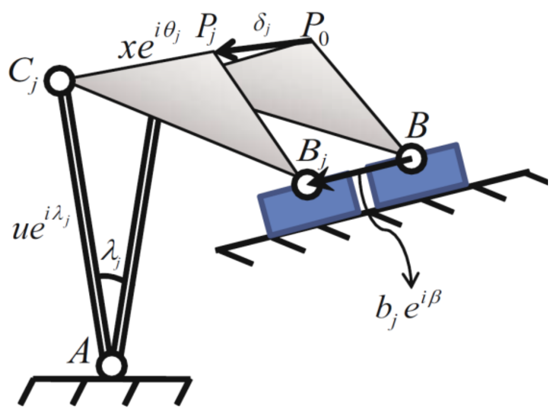
	n	Total Degree	GLP Bound	Root Count	Computation Cost
PPS (Plane)	6	64	10	10	Resultant
SS (Sphere)	7	128	20	20	Resultant
CS (Circular Cylinder)	8	16,384	2,184	804	POLSYS_GLP 2Hrs on PC
RPS (Hyperboloid)	10	262,144	9,216	1,024	POLSYS_GLP 11Hrs on PC
right RRS (Circular Torus)	10	1,048,576	868,352	94,622	POLSYS_GLP 72mins on BH
PRS (Elliptic Cylinder)	10	2,097,152	247,968	18,202	POLSYS_GLP 33min on BH
RRS (Torus)	12	4,194,304	448,702	42,786	POLSYS_GLP 42mins on BH

n: maximum number of task position
BH: Blue Horizon system of SDSC(1024 CPU used)
MPC: Beowulf cluster system of UCI medium performance computing
PC: Pentium 4 1.5 GHz
Hrs: CPU hours

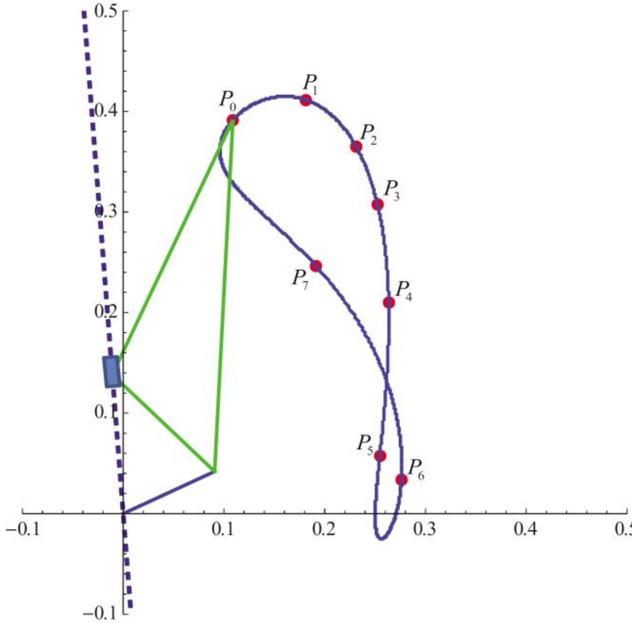
Eight Point Synthesis of Slider-Crank Linkages



(a)



(b)



$$
\begin{array}{l} f_1: \\ f_2: \\ f_3: \\ f_4: \end{array}
\begin{bmatrix}
(x-a)\lambda_j = x\theta_j - a + \delta_j \\
y\theta_j = b_j\beta + y - \delta_j \\
(\hat{x}-\hat{a})\hat{\lambda}_j = \hat{x}\hat{\theta}_j - \hat{a} + \delta_j^* \\
\hat{y}\hat{\theta}_j = b_j\hat{\beta} + \hat{y} - \delta_j^*
\end{bmatrix}, \quad j = 1, \ldots, n
$$

$$
\begin{array}{l} f_5: \\ f_6: \\ f_7: \end{array}
\begin{bmatrix}
\lambda_j\hat{\lambda}_j = 1 \\
\theta_j\hat{\theta}_j = 1 \\
\beta\hat{\beta} = 1
\end{bmatrix}, \quad j = 1, \ldots, n
$$

- 4n+1 equations with 5n+8 unknowns
- n=7 $\Rightarrow$ 43 quadratic polynomial
- Total degree=2^{43}=8.8x10^{12}
- Reduced to 26,880 paths with Bertini
- Total 558 solutions or **279** pairs of cognate slider-crank four-bar linkages

Eight Point Synthesis of Slider-Crank Linkages

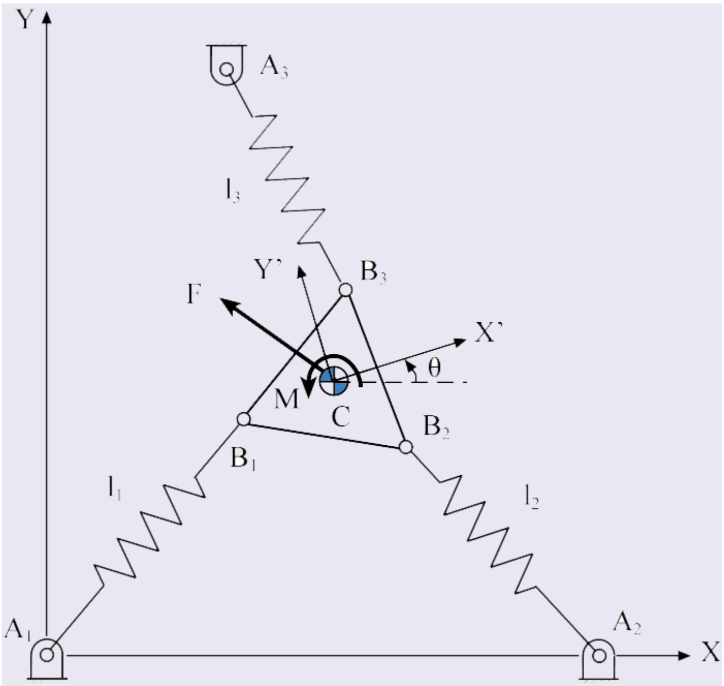
Table 1 Number of solutions to unaugmented and augmented systems, respectively, $P(z)=0$ and $\hat{P}(z,l)=0$

Method		p_{1-7}		p_{1-15}	
		No. of paths tracked	No. of solutions	No. of paths tracked	No. of solutions
BERTINI	Classical	26,880	14,582[a]	26,880	558
	Regeneration	14,576[a]	558	19,036[a]	558
HOM4PS2		5632	2348[b]	5632	558[c]

[a]This number slightly changes by $\pm 0.1\%$ depending on the input data.
[b]This number slightly changes by $\pm 1\%$ depending on the input data.
[c]This number slightly changes by -1% depending on the input data.

Kinetostatic Analysis of Compliant Mechanisms

PRBM of a compliant platform mechanism



- **Objective**
  - Find equilibrium configurations with given external forces
- **Inverse Static Analysis Equations**

$$l_i^2 = (\mathbf{B}_i - \mathbf{A}_i)^T(\mathbf{B}_i - \mathbf{A}_i), \quad i = 1, 2, 3.$$

Kinematics

$$\frac{\partial V}{\partial x} - F_x = \sum_{i=1}^{3} k_i(l_i - l_{0i})\frac{\partial l_i}{\partial x} - F_x = 0,$$

$$\frac{\partial V}{\partial y} - F_y = \sum_{i=1}^{3} k_i(l_i - l_{0i})\frac{\partial l_i}{\partial y} - F_y = 0,$$

$$\frac{\partial V}{\partial \theta} - M = \sum_{i=1}^{3} k_i(l_i - l_{0i})\frac{\partial l_i}{\partial \theta} - M = 0,$$

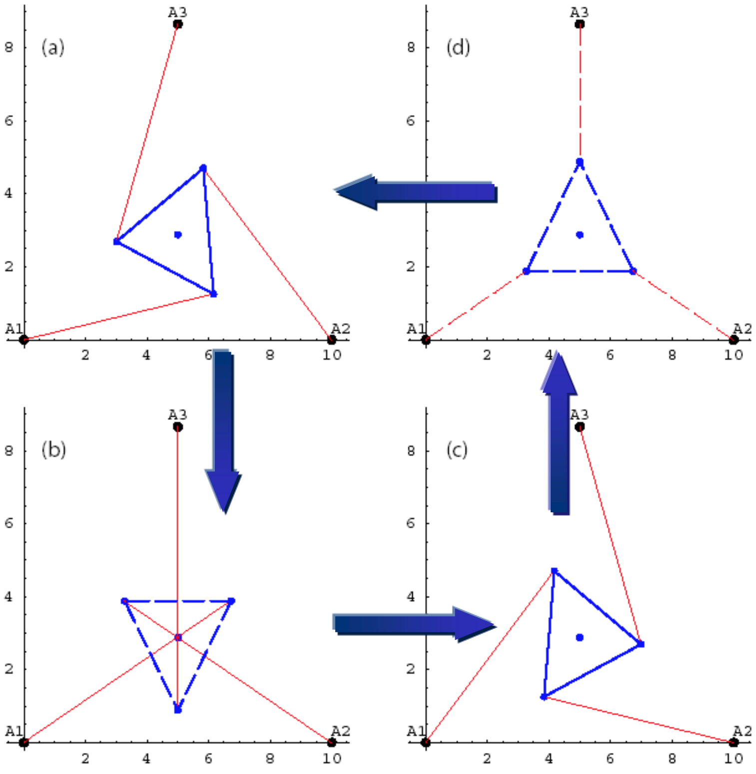
Equilibrium

V is the potential energy of the system.

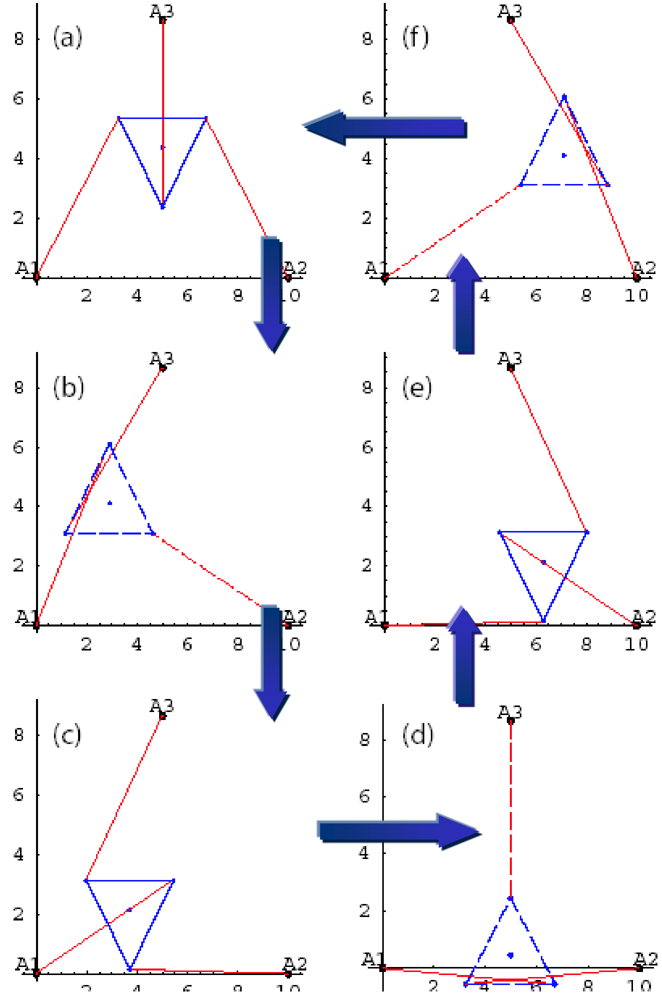
- **Solutions**
  - Apply POLSYS_GLP program
  - Find all equilibrium positions
  - Stability criteria: check definiteness of the Hessian matrix

Finding All Equilibrium Positions of CM

- **Zero external forces (unloaded)**
- **Symmetric base and platform**
- **POLSYS_GLP tracked 466 solution path**



A bi-stable compliant mechanism



A tri-stable compliant mechanism

Kinetostatic Analysis of Spatial Compliant Stewart-Gough Platform

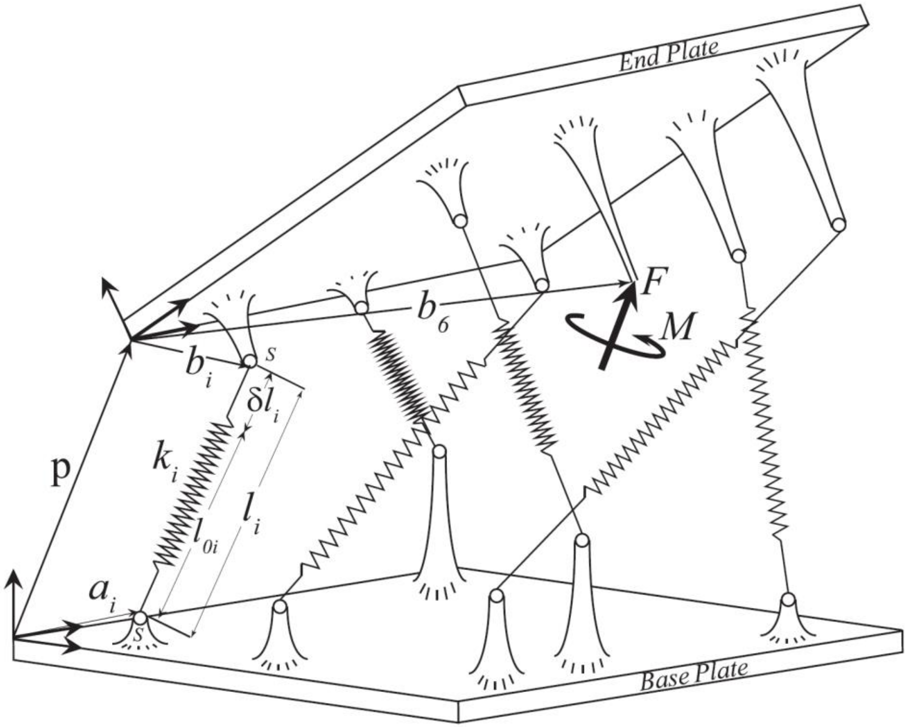


Fig. 1. A schematic view of a general compliant Stewart–Gough platform.

- **Kinematic constraint equations**

$$
\begin{aligned}
0 &= \hat{g}^T \hat{q}, \\
0 &= \hat{q}^T \hat{q} l_0^2 - \hat{g}^T \hat{g}, \\
0 &= \left[\mathbf{B}_j - \mathbf{a}_j\right]^T \left[\mathbf{B}_j - \mathbf{a}_j\right] - l_j^2, \quad j = 1, \ldots, 5 \\
&= 2\left(\tilde{\mathbf{g}}^T \mathbf{b}_j - \mathbf{a}_j^T \left[\mathbf{p} + R\mathbf{b}_j\right]\right) + \left(\mathbf{a}_j^T \mathbf{a}_j + \mathbf{b}_j^T \mathbf{b}_j + l_0^2 - l_j^2\right) \hat{q}^T \hat{q},
\end{aligned}
$$

- **Static equilibrium equations**

$$
\mathbf{F} = \sum_{i=0}^{5} \frac{f_i}{l_i} [\mathbf{B}_i - \mathbf{a}_i] = \frac{f_0}{l_0} \mathbf{p} + \sum_{j=1}^{5} \frac{f_j}{l_j} \left[\mathbf{p} - \mathbf{a}_j + R\mathbf{b}_j\right]
$$

$$
\begin{aligned}
\mathbf{M} + R\mathbf{b}_6 \times \mathbf{F} &= \sum_{j=1}^{5} \frac{f_j}{l_j} R\mathbf{b}_j \times \left[\mathbf{B}_j - \mathbf{a}_j\right] = \sum_{j=1}^{5} \frac{f_j}{l_j} R\mathbf{b}_j \times \left[\mathbf{p} - \mathbf{a}_j\right] \\
&= \sum_{j=1}^{5} \frac{f_j}{l_j} \left[G\mathbf{b}_j + \mathbf{a}_j \times R\mathbf{b}_j\right],
\end{aligned}
$$

Kinetostatic Analysis of Spatial Compliant Stewart-Gough Platform

- **13 polynomials**
- **Total degree = 5,971,968**
- **Bertini traced 253,602 paths**
- **Found 29,272 Solutions**

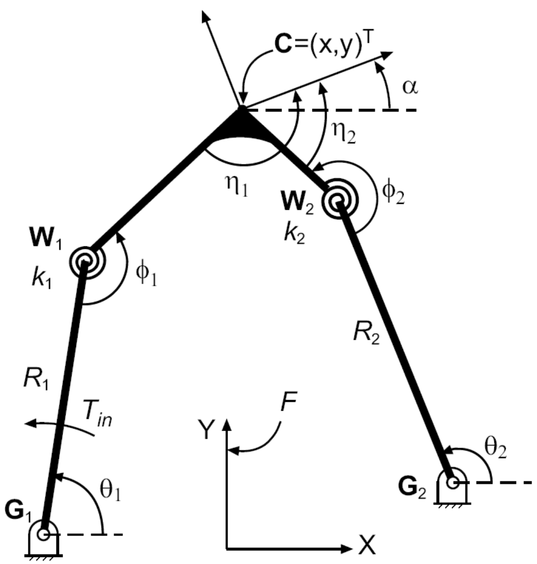
Table 1
Five kinetostatic problems of the compliant Stewart–Gough platform.

Kinetostatics problem type	Known parameters	Unknowns	Solution procedure: solve Eqs.	# of sols.
Inv.–Inv.	$\{\mathbf{l}_0, \mathbf{a}, \mathbf{b}, \hat{q}, \hat{g}, \mathbf{F}, \mathbf{M}\}$	$\{\mathbf{l}, \mathbf{k}\}$	(i) 14 (ii) 15,16	1
Inv.–Fwd.	$\{\mathbf{l}_0, \mathbf{a}, \mathbf{b}, \hat{q}, \hat{g}, \mathbf{k}\}$	$\{\mathbf{l}, \mathbf{F}, \mathbf{M}\}$	(i) 14 (ii) 15 (iii) 16	1
Fwd.–Inv.	$\{\mathbf{l}_0, \mathbf{l}, \mathbf{a}, \mathbf{b}, \mathbf{F}, \mathbf{M}\}$	$\{\hat{q}, \hat{g}, \mathbf{k}\}$	(i) 14 (ii) 15,16	40
Fwd.–Fwd.	$\{\mathbf{l}_0, \mathbf{l}, \mathbf{a}, \mathbf{b}, \mathbf{k}\}$	$\{\hat{q}, \hat{g}, \mathbf{F}, \mathbf{M}\}$	(i) 14 (ii) 15 (iii) 16	40
Coupled	$\{\mathbf{l}_0, \mathbf{a}, \mathbf{b}, \mathbf{F}, \mathbf{M}, \mathbf{k}\}$	$\{\mathbf{l}, \hat{q}, \hat{g}\}$	14,15,16	29,272

Tari, Su and Hauenstein, Mechanism and Machine Theory, 2012

Synthesis of Compliant Mechanisms

PRBM of a compliant 4-bar



- **Objective**
  - Find the dimensions and spring parameters for a given set of equilibrium positions
- **Synthesis Equations**
  - Write kinematics equations and equilibrium equations for each design position

$$\mathbf{W}_1^j - \mathbf{G}_1 - [R(\Delta\alpha^j - \Delta\phi_1^j)](\mathbf{W}_1^0 - \mathbf{G}_1) = 0, \quad j = 1,2,$$

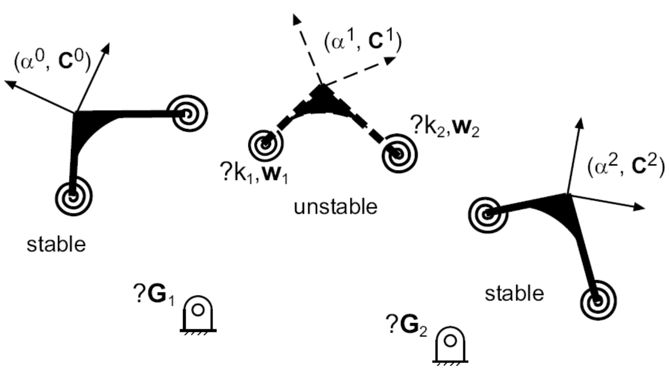
$$\mathbf{W}_2^j - \mathbf{G}_2 - [R(\Delta\alpha^j - \Delta\phi_2^j)](\mathbf{W}_2^0 - \mathbf{G}_2) = 0, \quad j = 1,2.$$

$$(\mathbf{W}_1^j - \mathbf{W}_2^j)\nu_1^j - (\mathbf{W}_1^j - \mathbf{G}_1) + (\mathbf{W}_2^j - \mathbf{G}_2)\nu_2^j = 0, \quad j = 1,2,$$

Kinematics

$$k_1\Delta\phi_1^j(\nu_1^j - 1) + k_2\Delta\phi_2^j(\nu_1^j - \nu_2^j) = 0, j = 1,2,$$

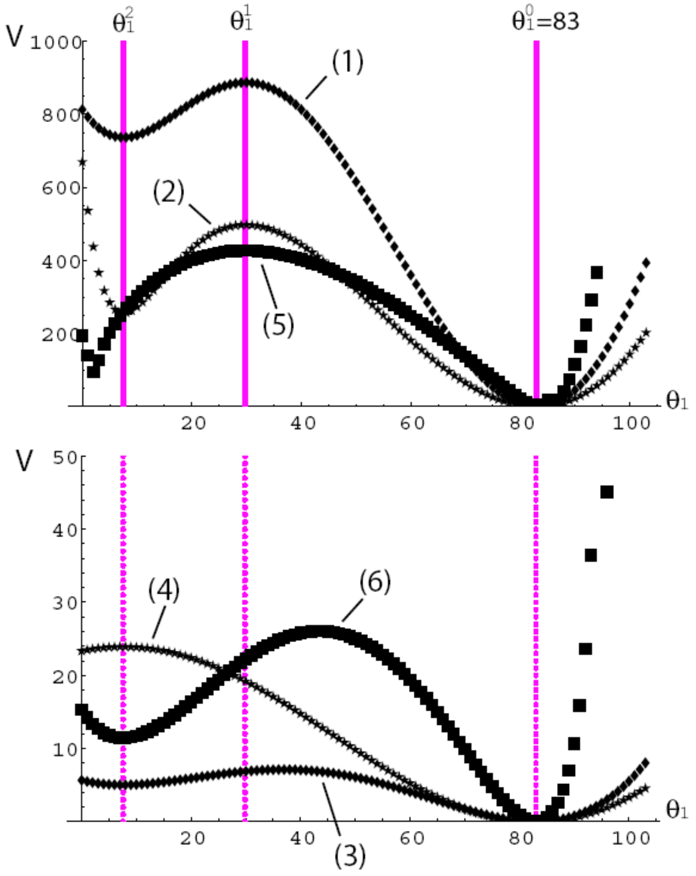
Equilibrium



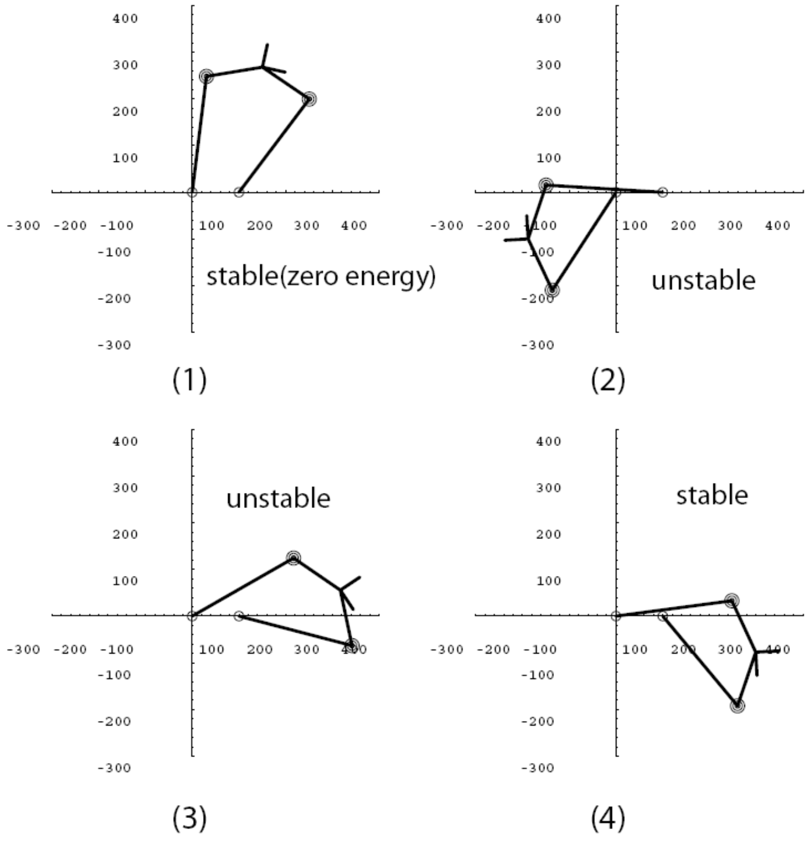
- **Solutions**
  - Transform equations into polynomial form
  - Solved by POLSYS_GLP program
  - Find all candidate designs

Synthesis of Bistable Compliant Mechanisms

- **POLSYS_GLP tracked 196 solution path**
- **Found 8 real solutions**



Analysis of solution #1



Kinetostatic Synthesis of Compliant 4-Bar

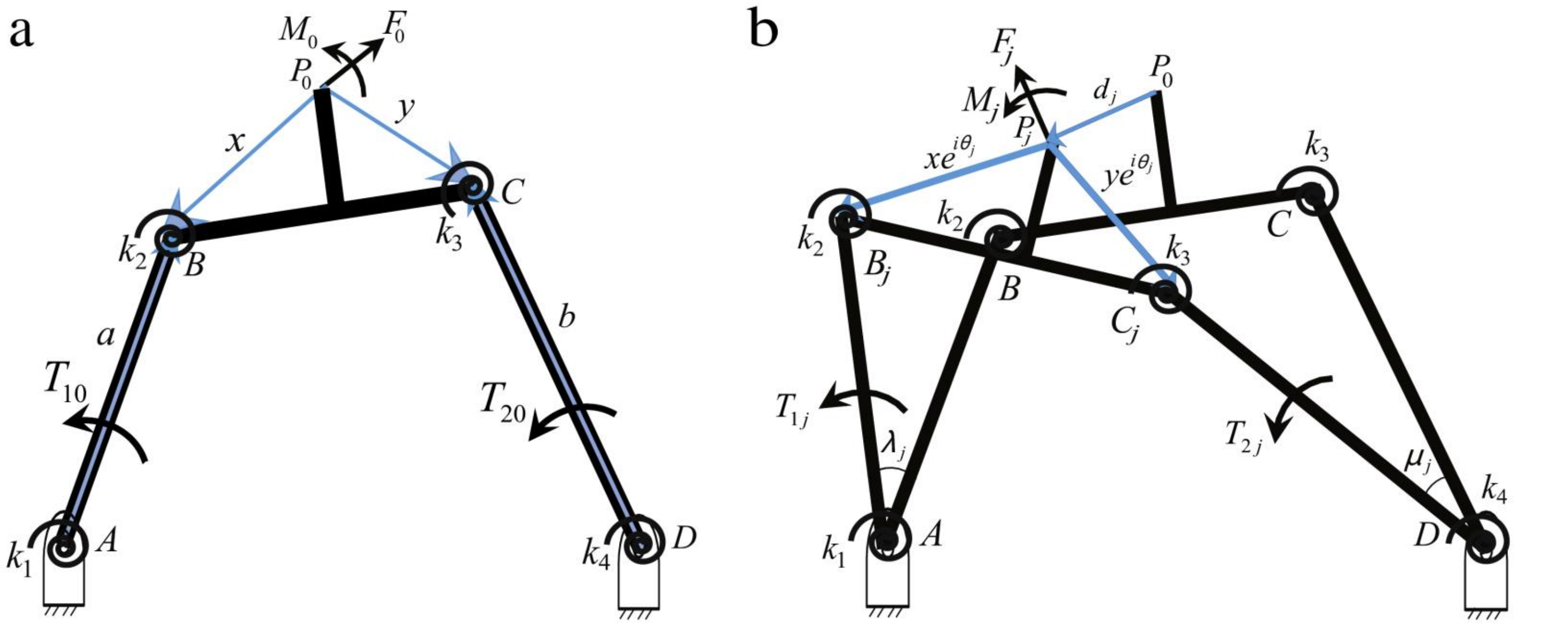


Fig. 2. A schematic view of a pseudo-rigid-body four-bar and its displaced configuration with the vector definitions.

- **Kinematic constraint equations**

$$
\left.\begin{aligned}
f_1 &: \mathbf{a}\left(e^{i\lambda_j}-1\right)-\mathbf{x}\left(e^{i\theta_j}-1\right)=\mathbf{d}_j \\
f_2 &: \mathbf{b}\left(e^{i\mu_j}-1\right)-\mathbf{y}\left(e^{i\theta_j}-1\right)=\mathbf{d}_j
\end{aligned}\right., \quad \mathrm{j}=1,\ldots,\mathrm{n},
$$

$$
\left.\begin{aligned}
f_3 &: \mathbf{a}\left(e^{-i\lambda_j}-1\right)-\hat{\mathbf{x}}\left(e^{-i\theta_j}-1\right)=\mathbf{d}_j \\
f_4 &: \hat{\mathbf{b}}\left(e^{-i\mu_j}-1\right)-\hat{\mathbf{y}}\left(e^{-i\theta_j}-1\right)=\hat{\mathbf{d}}_j
\end{aligned}\right., \quad \mathrm{j}=1,\ldots,\mathrm{n},
$$

- **Static equilibrium equations**

$$
f_5 : \begin{vmatrix}
\mathbf{a}\, e^{i\lambda_j} & -\mathbf{b}\, e^{i\mu_j} & (\mathbf{y}-\mathbf{x})e^{i\theta_j} \\
\hat{\mathbf{a}}\, e^{-i\lambda_j} & -\hat{\mathbf{b}}\, e^{-i\mu_j} & (\hat{\mathbf{y}}-\hat{\mathbf{x}})e^{-i\theta_j} \\
c_{1j} & c_{2j} & c_{3j}
\end{vmatrix} = 0
$$

Enumeration of Kinetostatic Problems

Function Generation

Table 1
All combinations of free choices for general compliant function generation problems.

Case	n	# of eqs. ($3n$)	$m_1 \leq 6-n$	$k \leq 4$	# of sols. $O(\infty^{10-2n})$
1	5	15	0	0	Finite
2	4	12	0	2	$O(\infty^2)$
3	4	12	1	1	$O(\infty^2)$
4 [a]	4	12	2	0	$O(\infty^2)$
5	3	9	0	4	$O(\infty^4)$
6	3	9	1	3	$O(\infty^4)$
7	3	9	2	2	$O(\infty^4)$
8 [a]	3	9	3	1	$O(\infty^4)$
9	2	6	2	4	$O(\infty^6)$
10	2	6	3	3	$O(\infty^6)$
11 [a]	2	6	4	2	$O(\infty^6)$
12 [a]	1	3	4	4	$O(\infty^8)$

[a] Cases with decoupled kinematic and static equations.

Path Generation

Table 3
All combinations of free choices for general compliant path generation problems.

Case	n	# of eqs. ($5n$)	$m_4 \leq 8-n$	$k \leq 4$	# of sols. $O(\infty^{12-2n})$
27	6	30	0	0	Finite
28	5	25	0	2	$O(\infty^2)$
29	5	25	1	1	$O(\infty^2)$
30	5	25	2	0	$O(\infty^2)$
31	4	20	0	4	$O(\infty^4)$
32	4	20	1	3	$O(\infty^4)$
33	4	20	2	2	$O(\infty^4)$
34	4	20	3	1	$O(\infty^4)$
35 [a]	4	20	4	0	$O(\infty^4)$
36	3	15	2	4	$O(\infty^6)$
37	3	15	3	3	$O(\infty^6)$
38	3	15	4	2	$O(\infty^6)$
39 [a]	3	15	5	1	$O(\infty^6)$
40	2	10	4	4	$O(\infty^8)$
41	2	10	5	3	$O(\infty^8)$
42 [a]	2	10	6	2	$O(\infty^8)$
43	1	5	6	4	$O(\infty^{10})$
44 [a]	1	5	7	3	$O(\infty^{10})$

[a] Cases with decoupled kinematic and static equations.

Motion Generation

neral compliant motion generation problems.

# of eqs. ($3n$)	$m_2 \leq 4-n$	$m_3 \leq 4-n$	$k \leq 4$	# of sols. $O(\infty^{12-3n})$
12	0	0	0	Finite
9	0	0	3	$O(\infty^3)$
9	0	1	2	$O(\infty^3)$
9	1	0	2	$O(\infty^3)$
9	1	1	1	$O(\infty^3)$
6	0	2	4	$O(\infty^6)$
6	1	1	4	$O(\infty^6)$
6	1	2	3	$O(\infty^6)$
6	2	0	4	$O(\infty^6)$
6	2	1	3	$O(\infty^6)$
6	2	2	2	$O(\infty^6)$
3	2	3	4	$O(\infty^9)$
3	3	2	4	$O(\infty^9)$
3	3	3	3	$O(\infty^9)$

ıd static equations.

Tari and Su, Mechanism and Machine Theory, 2011

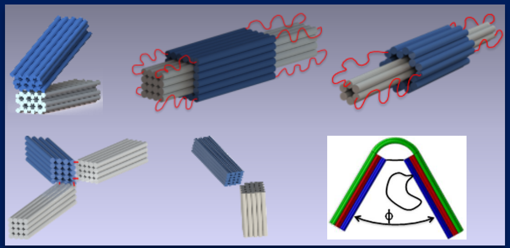
DNA Origami Nanorobots (Overview)

Background

- Emerging DNA origami nanotechnology
- Engineering complex molecular structures via self-assembling
- Current research mostly focuses on static structures
- Need for a systematic design approach for dynamic structures

Approach

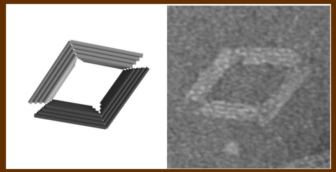
- Apply kinematic principle and theories from macroscopic engineering
- Design of various kinematic joints
- Computer-aided design/engineering

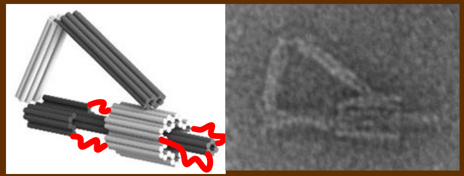


Design, Fabrication and Actuation of DNA Origami Nanomechanisms

Results

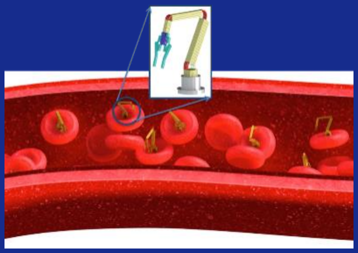
- Examples: Bennett 4-bar, slider-crank, compliant bistable 4-bar
- Design tools: projection kinematic analysis, design automation

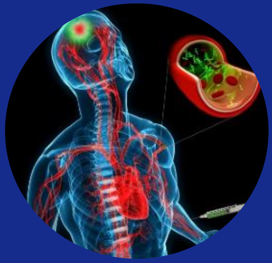




Applications

- Drug delivery
- Precision medicine
- Sensors for diagnose

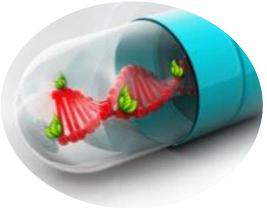




Nanotechnology for Precision Medicine

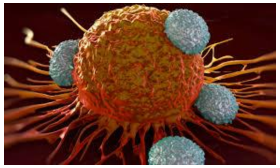
Drug delivery devices

deliver medicine to targeted area



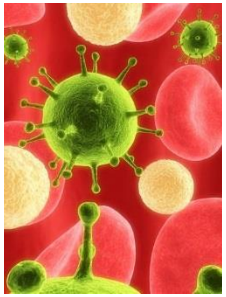
Therapeutics for treating cancers

targeted killing cancer cells



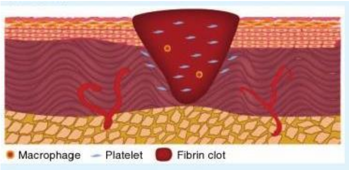
Non-invasive devices for diagnose

increase the efficiency and accuracy of diagnosis



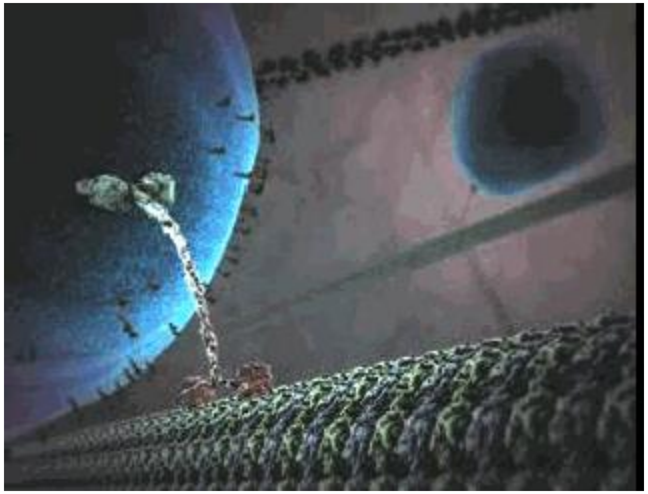
Tissue repair & self healing

reproduce or repair damaged tissue/cells

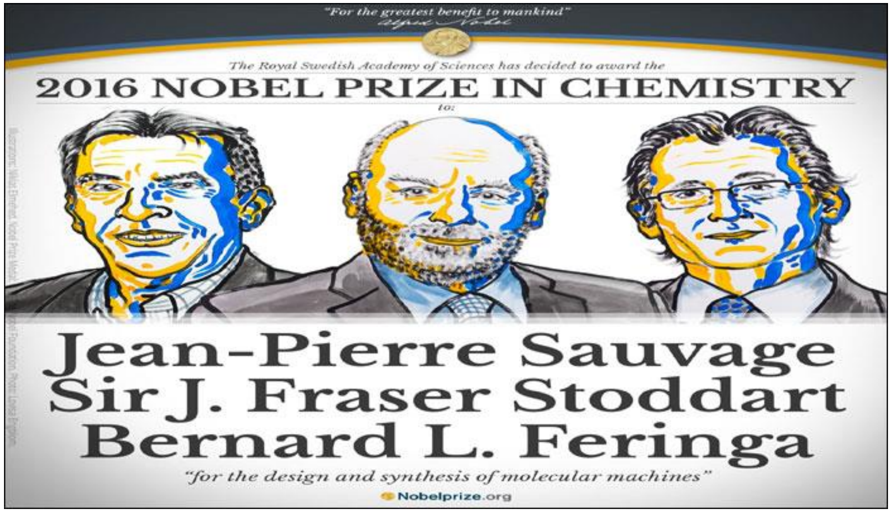


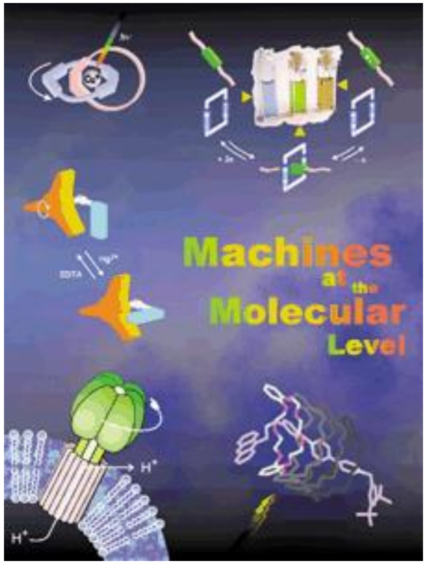
Engineering Molecular Machines

- **Why nanomachines and nanorobots?**
  - Needs: transportation, mechanical work
  - Programmable: prescribed motion
  - Controllable motion: reversible, responsive to actuation signal
- **Artificial Molecular Machines (2016 Nobel Prize in Chemistry**



The Inner Life of the Cell (Youtube)

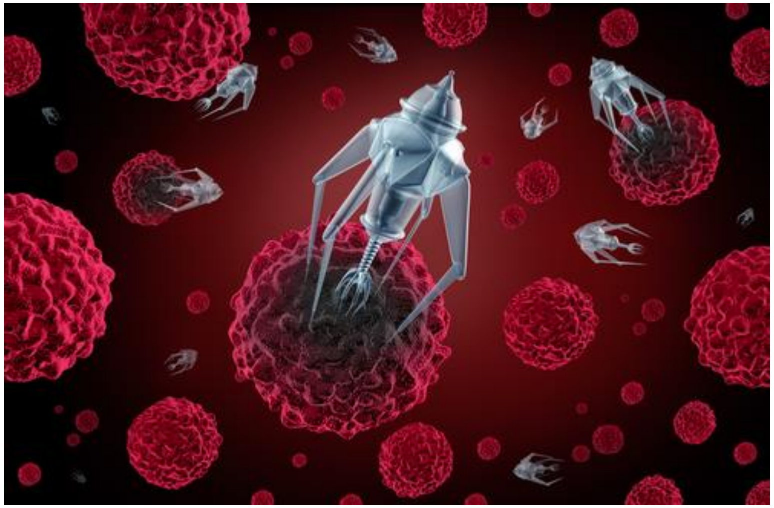


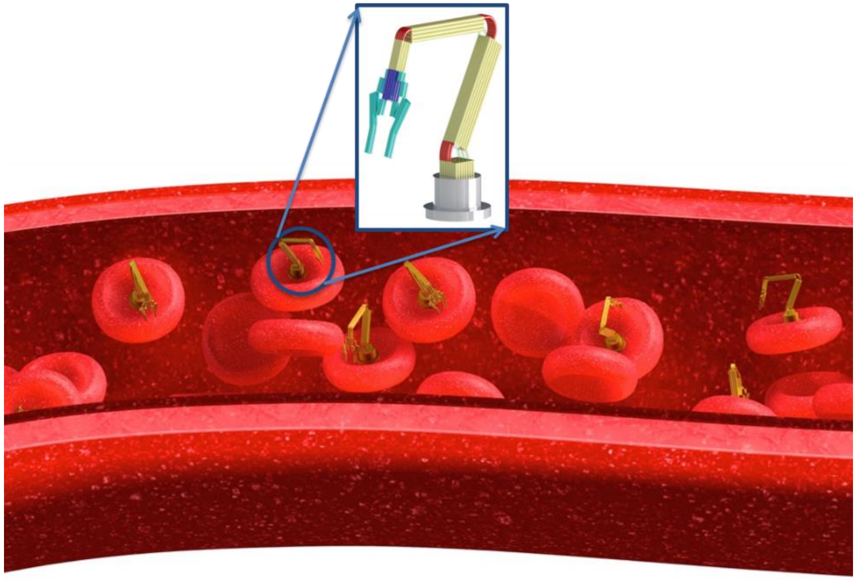


Artificial molecular machines, (V. Balzani, A. Credi, F. M. Raymo, and J. F. Stoddart), Angew. Chem. Int. Ed. 2000, 39, 3348-3391.

Research Goal

- **Apply mechanical engineering design principles to engineer/make molecular machines which could operate or re-order matter at a molecular or atomic scale in a controllable, programmable and pre-defined way.**

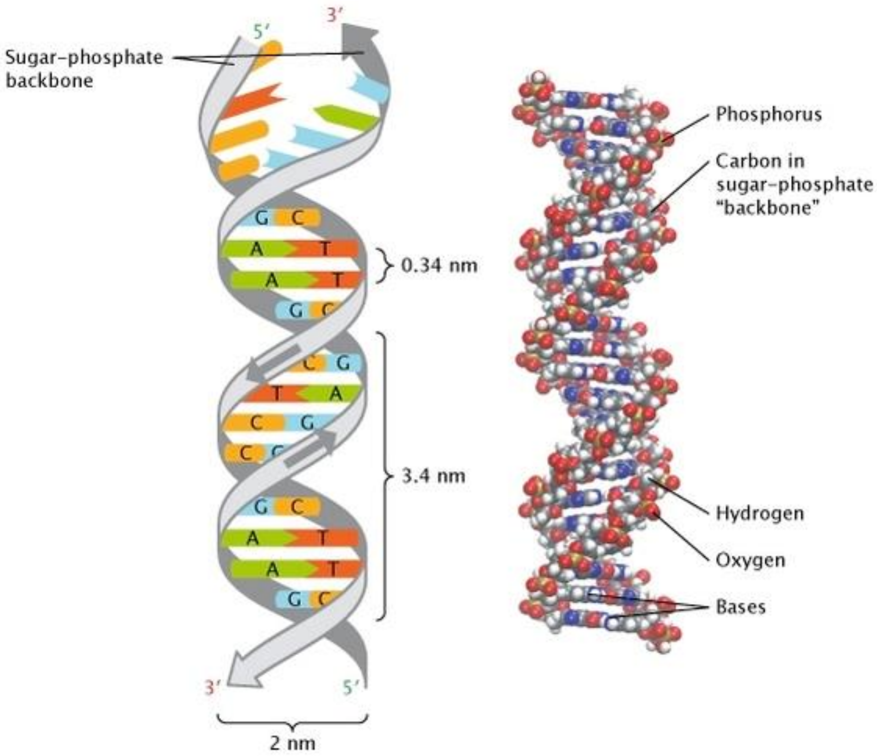




Su and Castro, The rise of DNA nanorobots, ASME Magazine, 2016

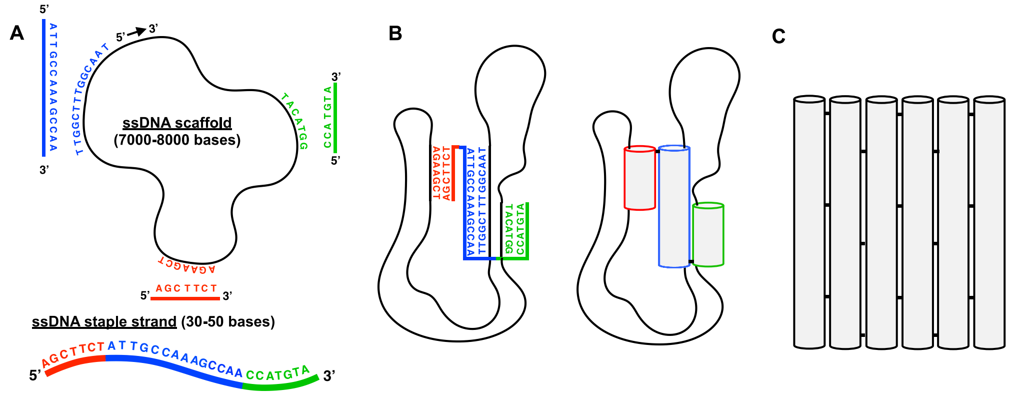
DNA Structures and Base Pairing

- **A long chain of four nucleotide acids**
  - adenine (A), thymine (T), (C) and guanine (G)
- **Single stranded DNA polymer (ssDNA)**
- **Base pairing**
  - A-T, C-G base paring
- **Self-assembling into double stranded helix structure (dsDNA)**

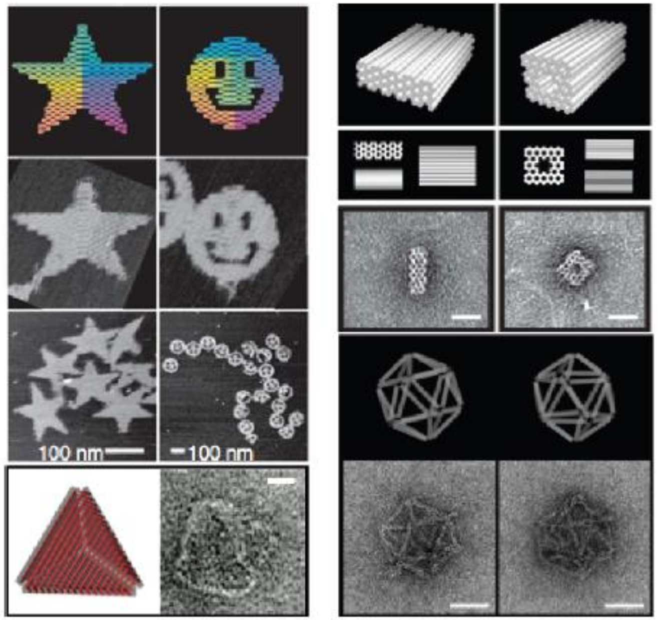


DNA Origami Nanotechnology

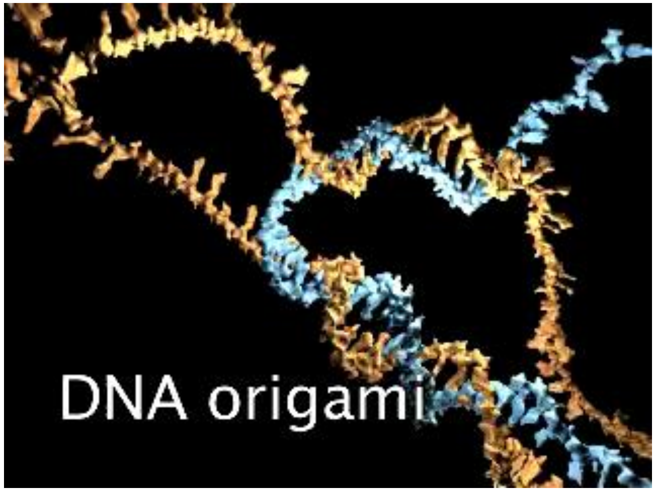
- **DNA Nanotechnology**
  - Design of complex structures using DNA materials
  - Fabrication via DNA self-assembling, i.e. A-T, C-G base paring



DNA origami nanotechnology for assembling complex structures



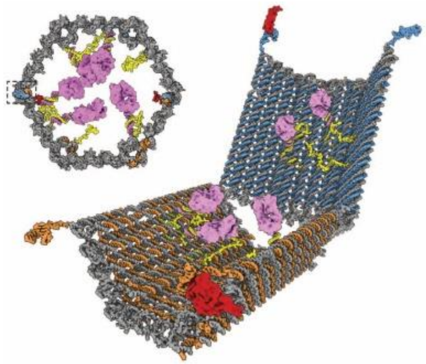
Rothemund, Nature, 2006



Dynamic Structures Made by DNA Origami Nanotechnology

- **Current designs are mostly static shapes**
- **Very few dynamic structures with controllable, programmable motions**
- **Our approach:**
  - Apply engineering principles to guide the design process
  - Apply kinematic and machine theory to design molecular machines for a prescribed motion
  - Actuate these molecular machines using staple displacement method or changing experimental settings.

Logic-gated nanorobot for drug delivery

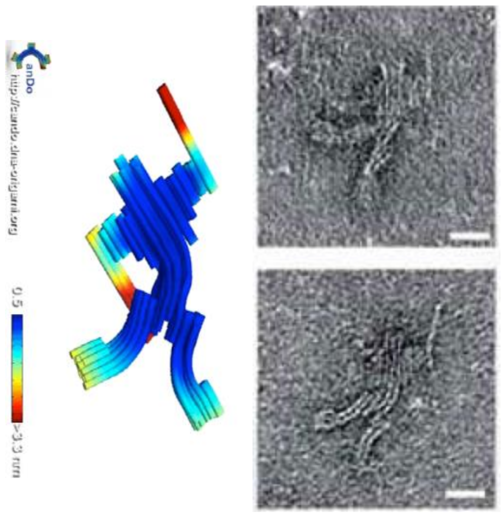


DNA origami forms a hexagonal barrell split in half

Complementary clamps close the barrell

Binding sites inside the barrel can be loaded with cargo conjugated to ssDNA

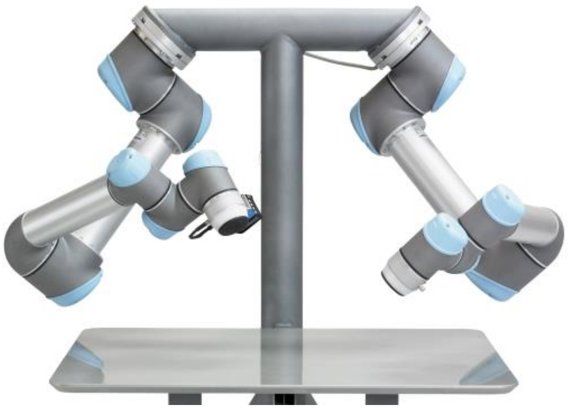
Douglas et al., Science 2012

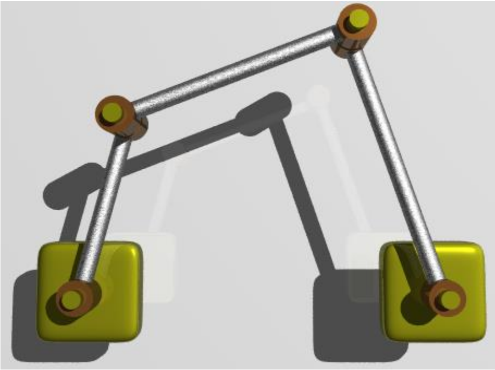


Castro et al., Nat. Meth. 8:221 (2011)

Kinematic Mechanisms and Machines

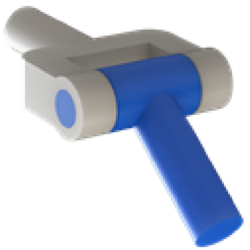
- (Kinematic) mechanisms are mechanical skeleton of machines and robots
- Mechanisms are comprised of multiple links connected by kinematic joints
- Mechanisms have a prescribed motion defined by connectivity of links and joints
- Their motion can be controlled via actuators

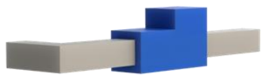


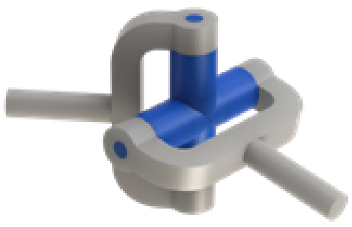


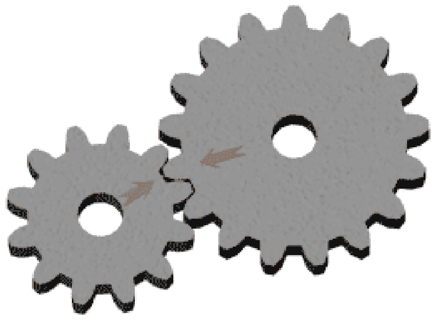
Kinematic Joints/Pairs

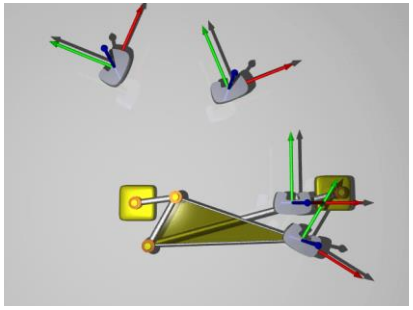
- Kinematic joints constrains motion of two connected parts
- Kinematic pairs: lower pairs, gear pairs, cams

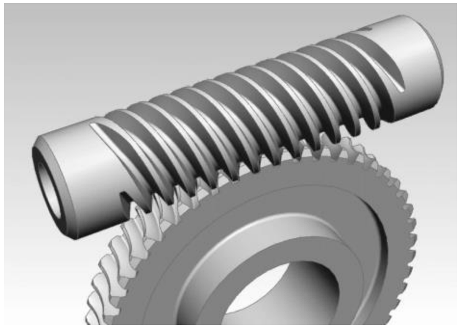


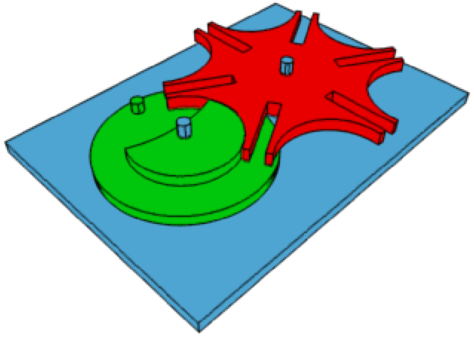


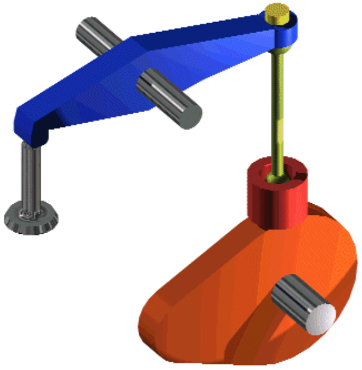








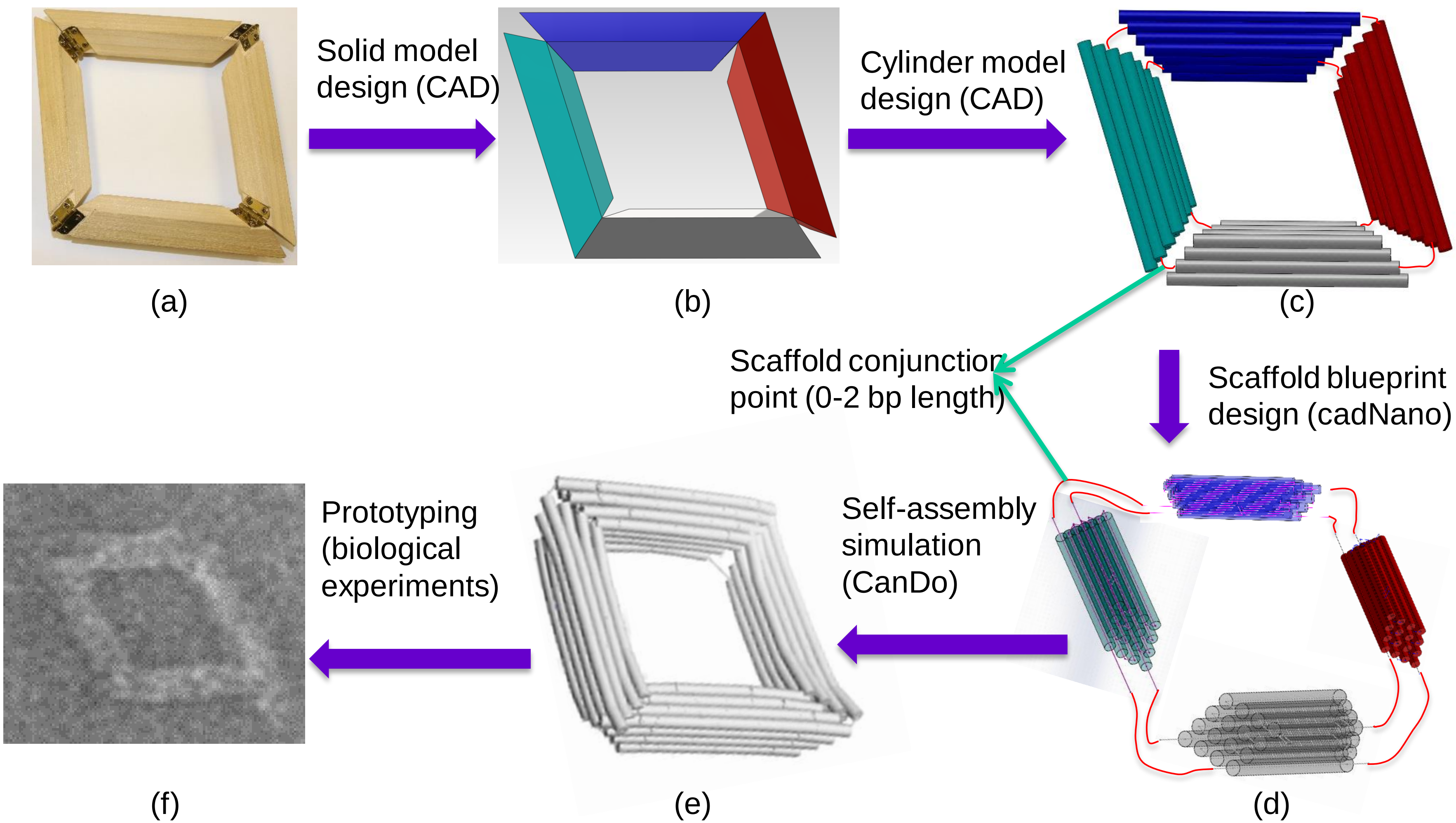




Design Process of Macroscopic Kinematic Mechanisms

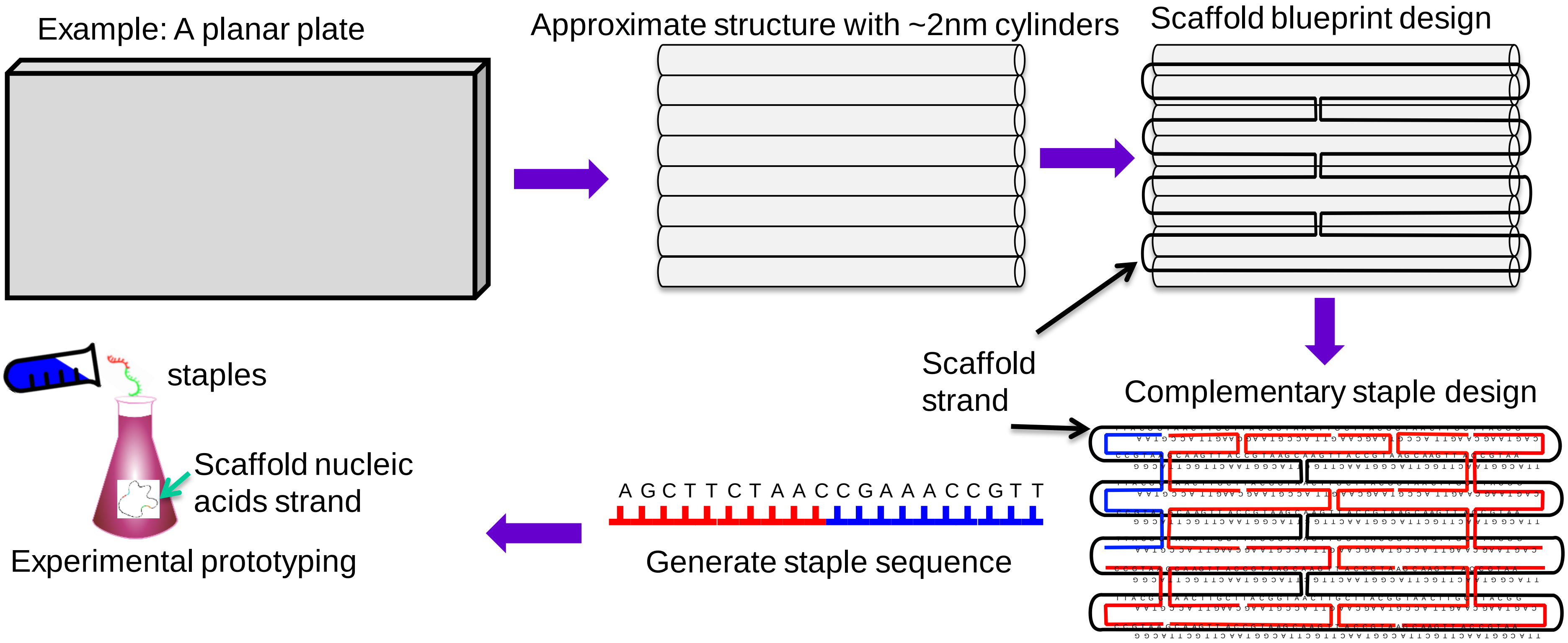
- **Number synthesis determines**
  - Number of links and joints
  - Joint types: R/P/C/S/U
  - Mobility (degree-of-freedom) of the final mechanism
- **Type synthesis determines**
  - How links and joined are connected. Single or multi loop
  - The motion characteristics of the final mechanism
- **Dimensional synthesis determines**
  - The critical kinematic parameters, e.g. link lengths, twist angles, location of joints
- **Embodiment design determines**
  - The dimensions of link shape

Design Process of DNA Origami Mechanisms



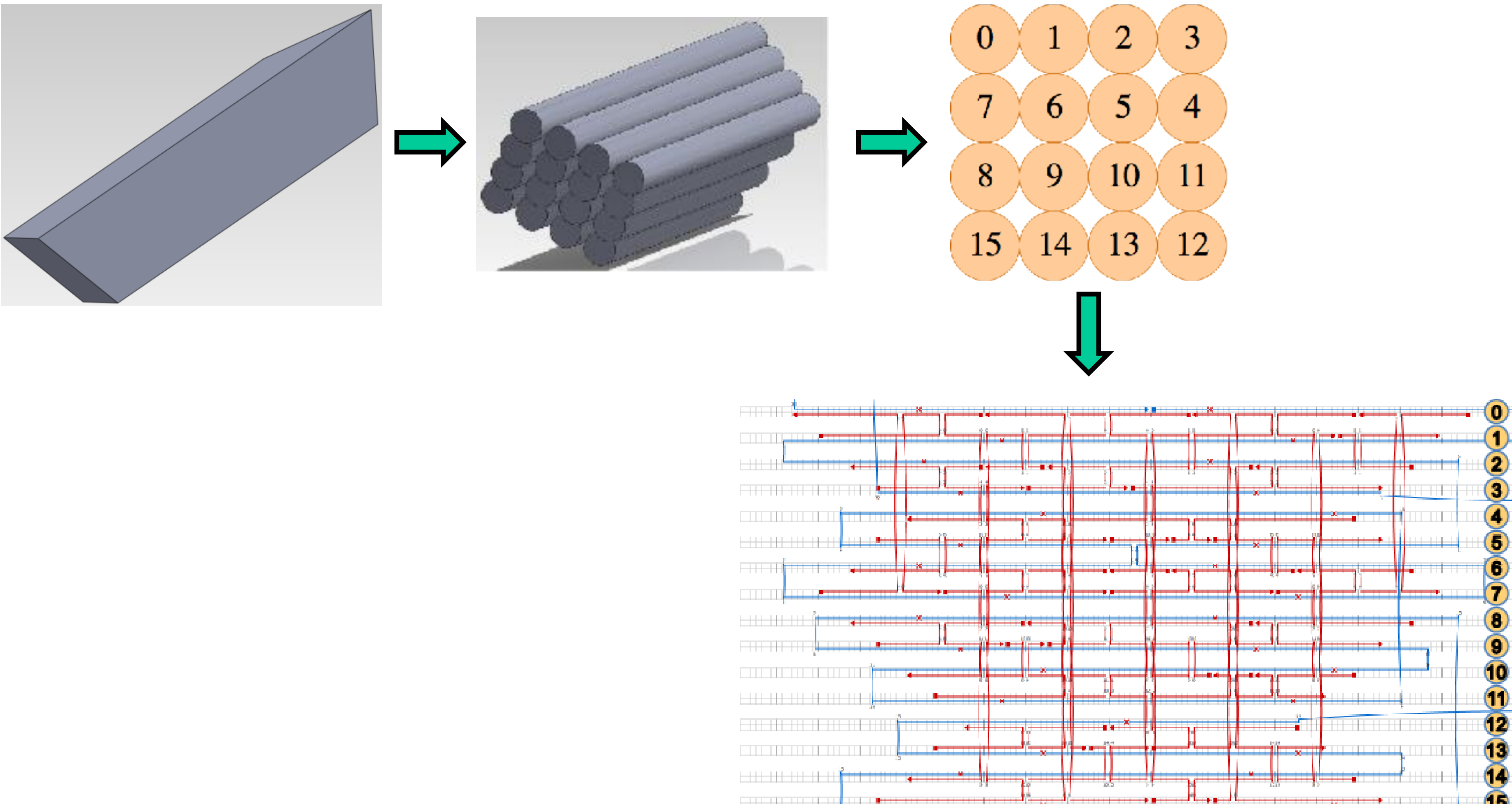
Design Process of DNA Links

- **Links are designed by bundles of double stranded DNA helices (dsDNA)**
- **Blueprint of scaffold design**
- **Staple sequence design**

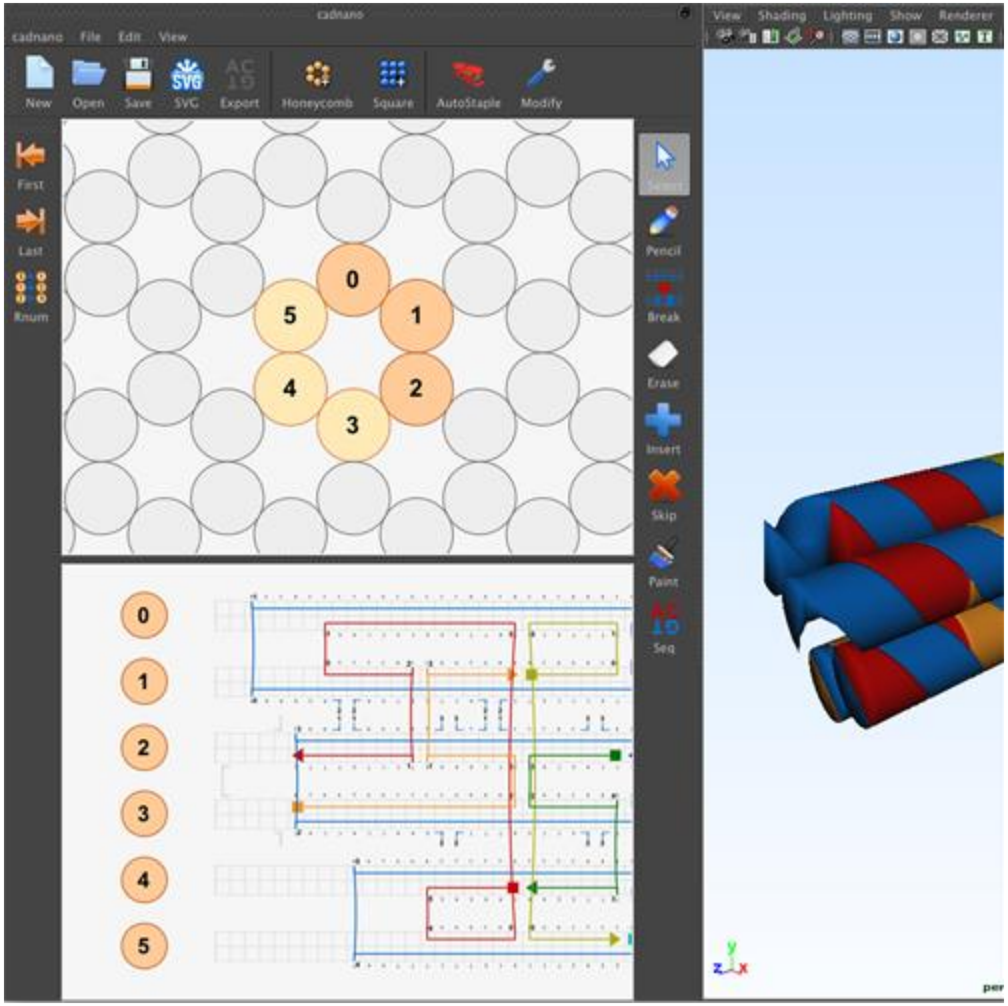


Design of DNA Origami Links

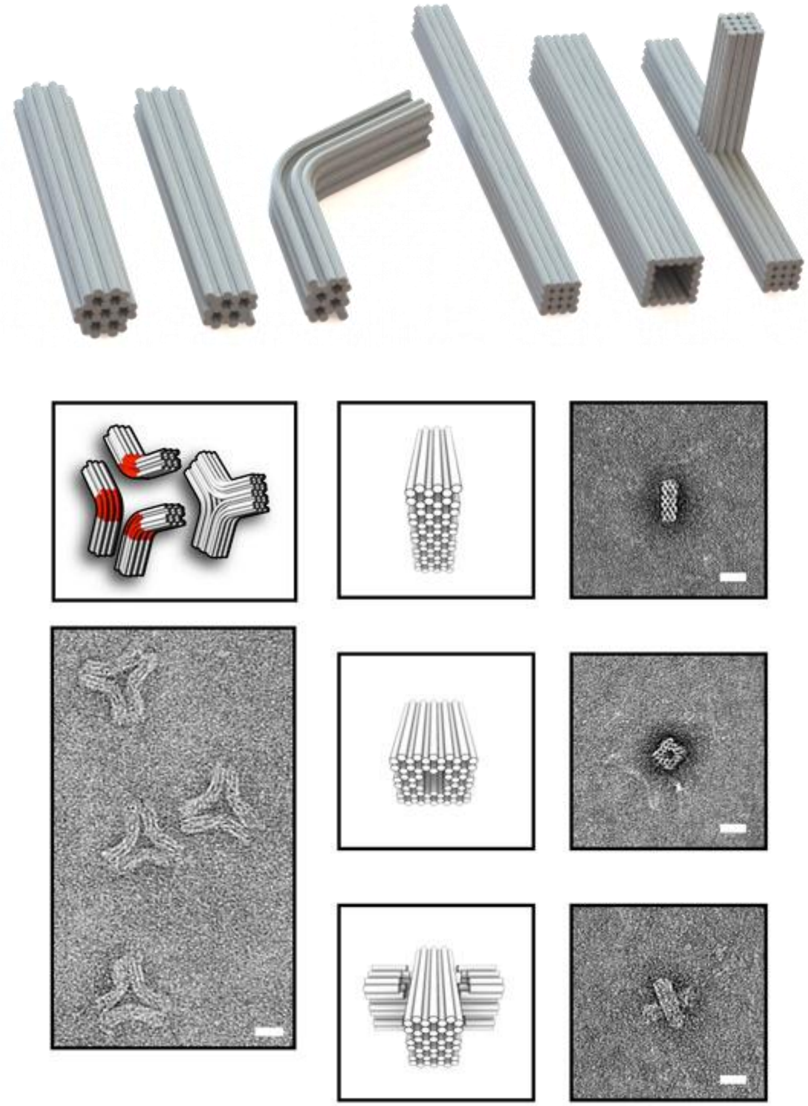
- **Use different length cylinders to approximate the desired shape**



Link Design with cadnano

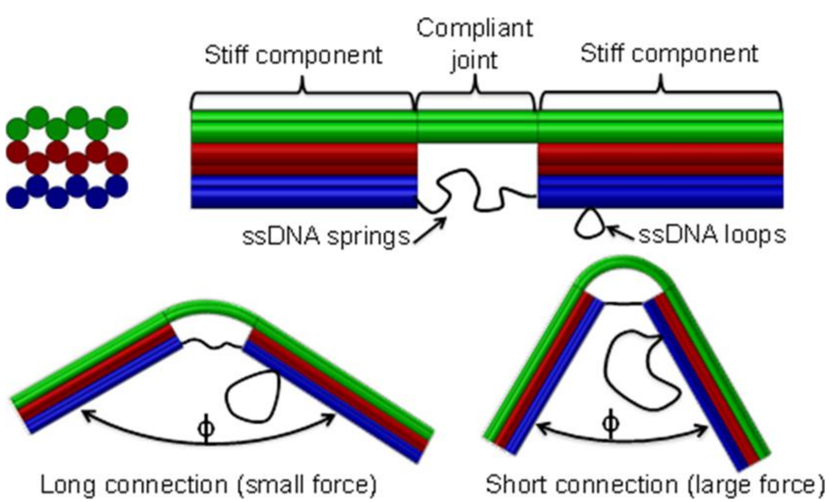


cadnano GUI

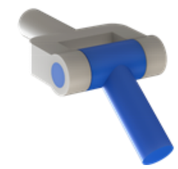
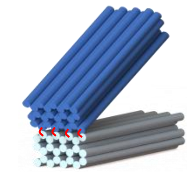
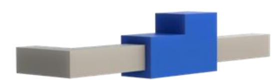
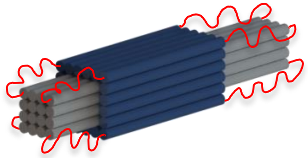
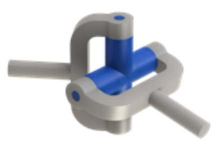
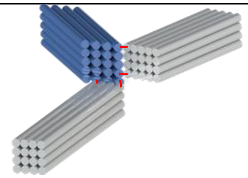
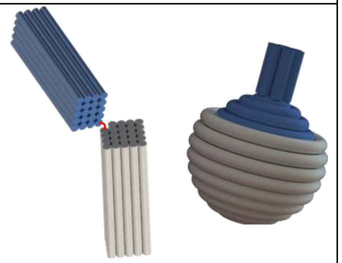


Design of Various DNA Joints

- **Kinematic joints are made of single stranded DNA**
- **Compliant joints are made of dsDNA with a small bending stiffness**

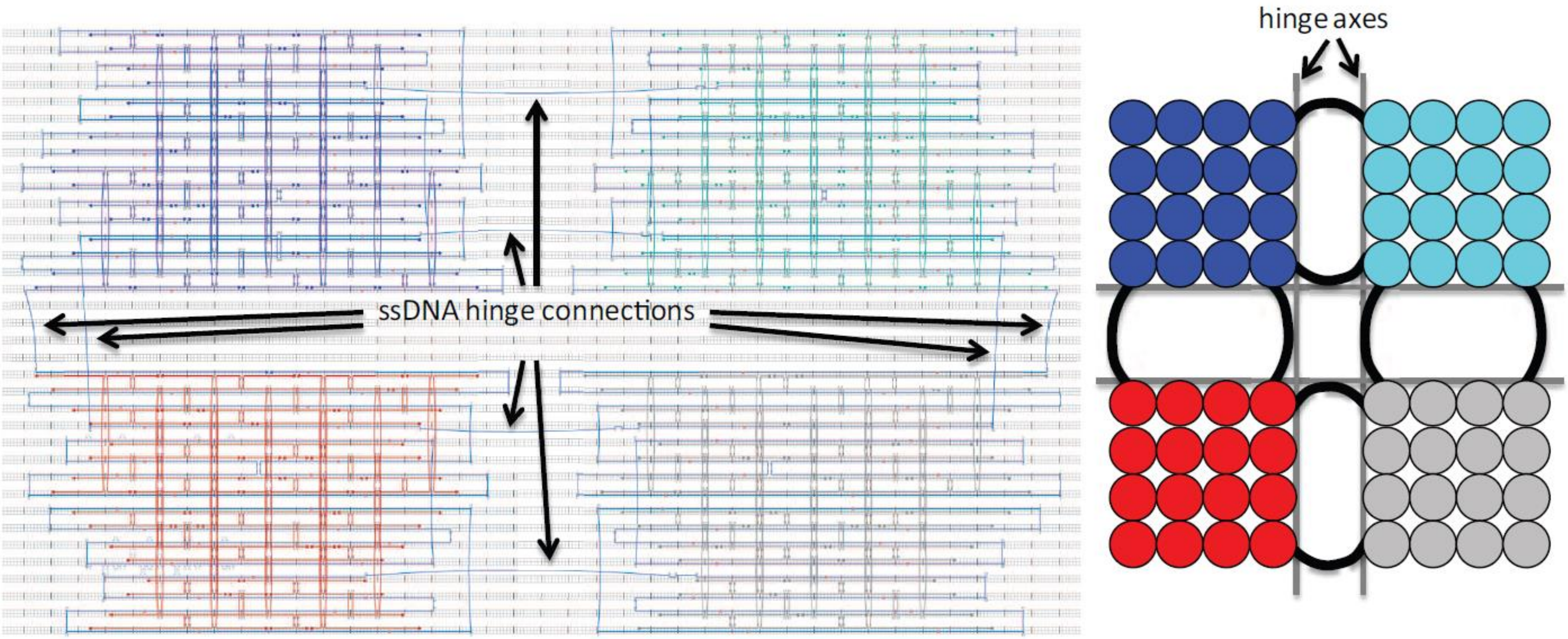


A design of compliant DNA joint

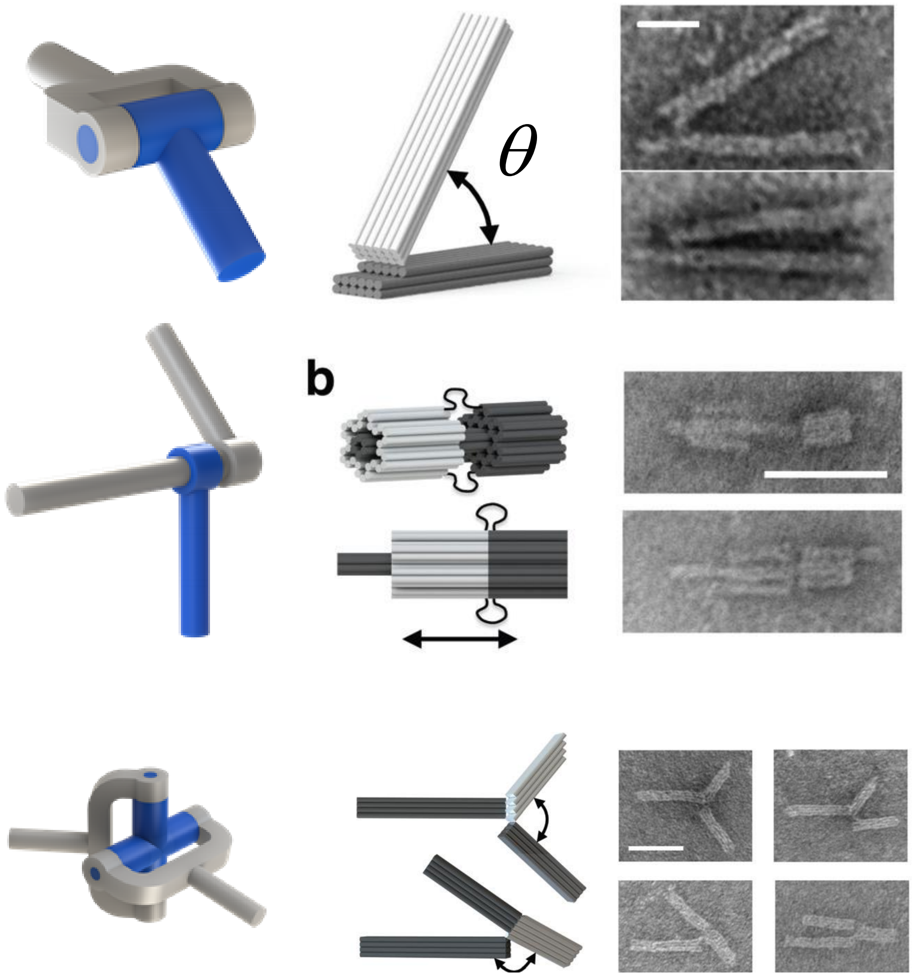
Joint	DOF	Solid Model	DNA Origami Design
Revolute	1		
Prismatic	1		
Cylindrical	2		
Universal	2		
Spherical	3		

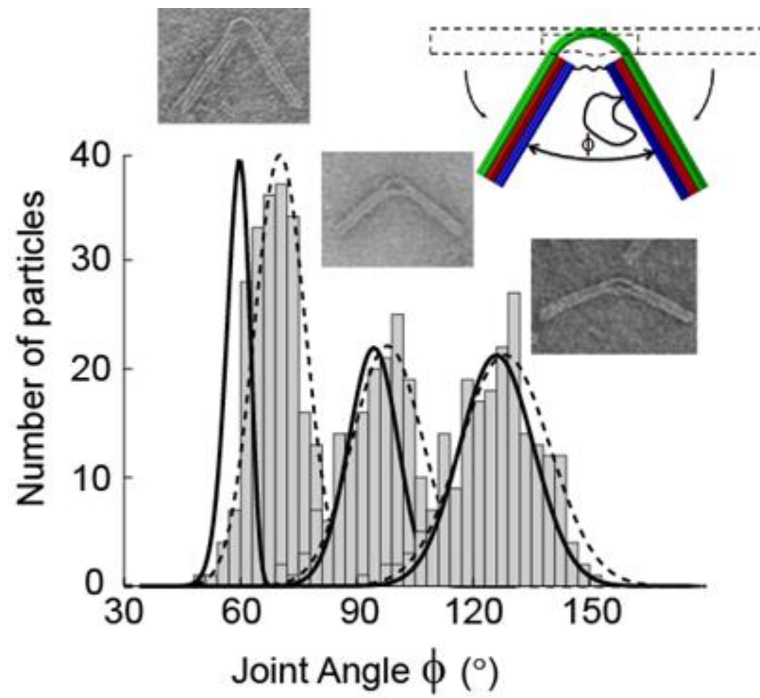
Design of DNA Origami Joints

- **Hinge design with ssDNA**



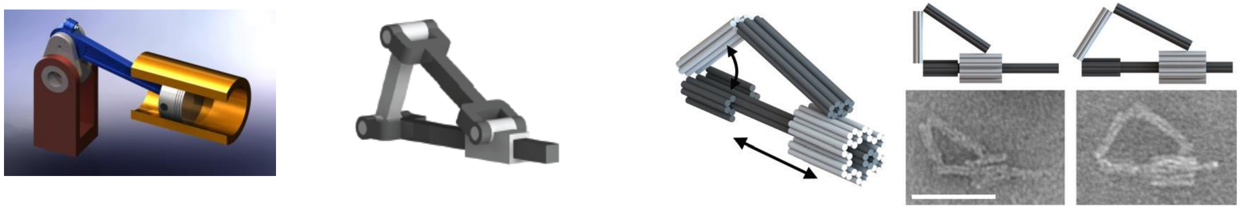
Experimental Results



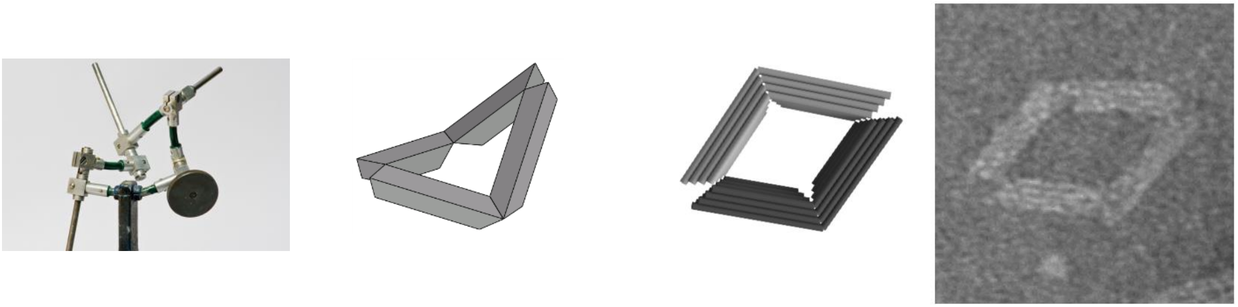


DNA Origami Mechanism Examples

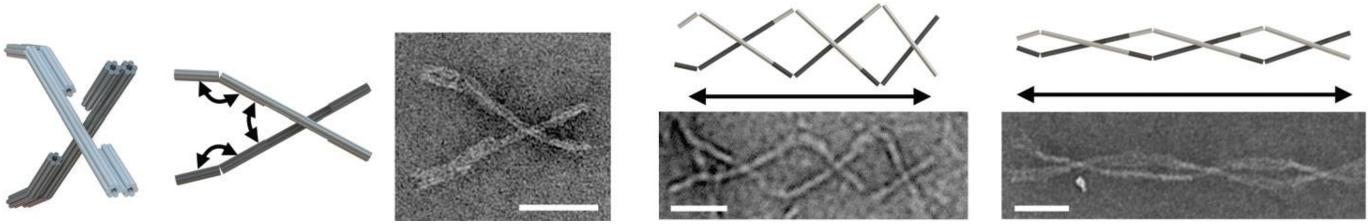
Crank-Slide Mechanism



Bennett Linkage



Scissors Linkage

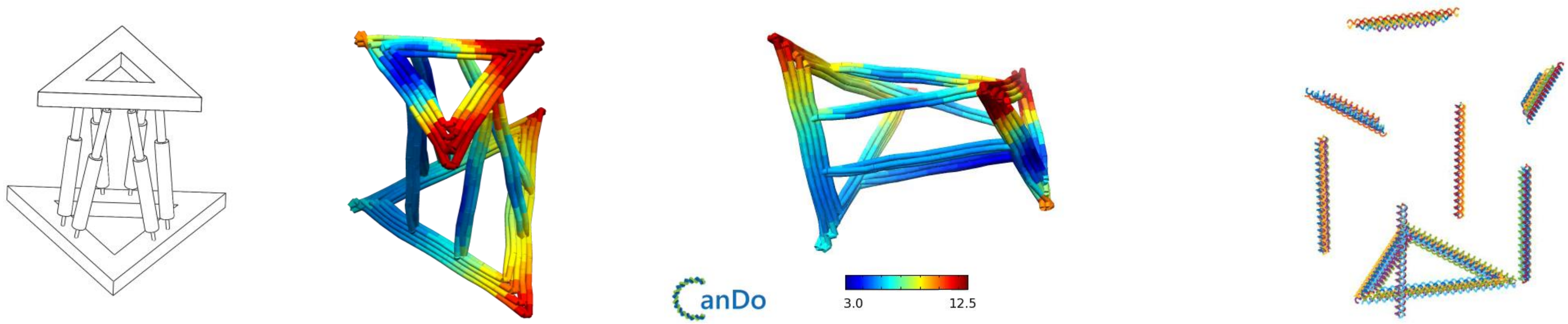


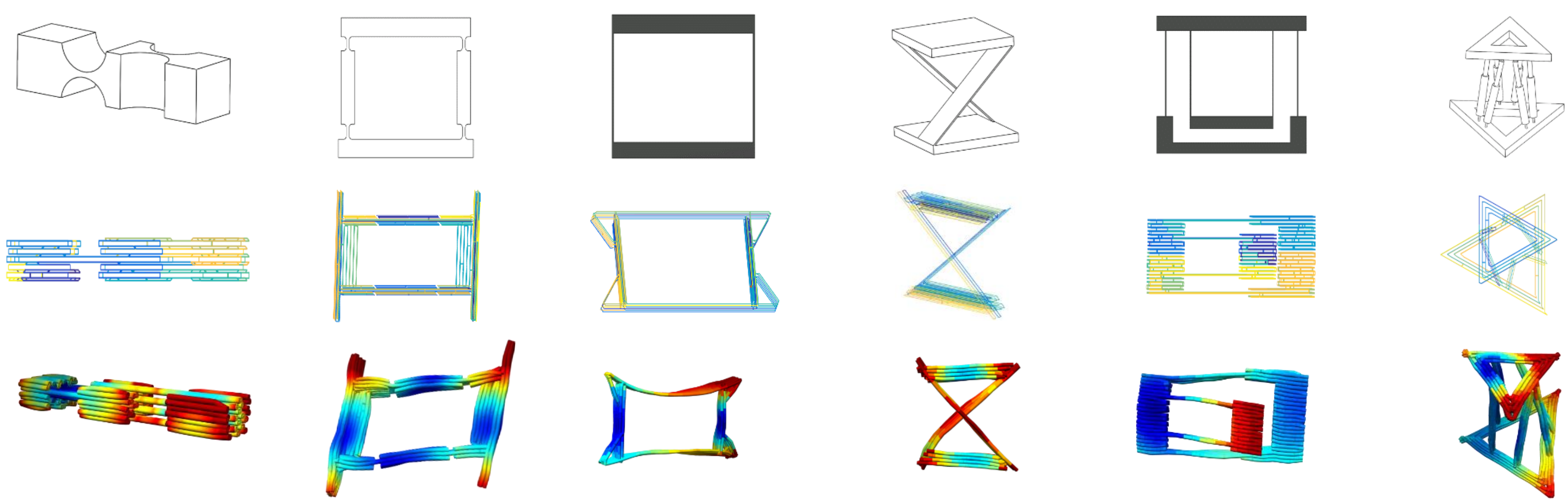
Castro, C. E.; Su, H.-J.; Marras, A. E.; Zhou, L.; Johnson, J. *Nanoscale* **2015**.

Marras, A. E.; Zhou, L.; Su, H.-J.; Castro, C. E. *Proc. Natl. Acad. Sci.* **2015**, *112*, 713–718.

Computer-Aided Design and Engineering

- **Design and CanDo simulation of Stewart Platform Linkage**

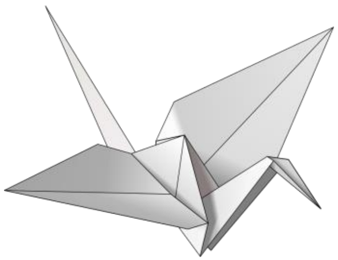


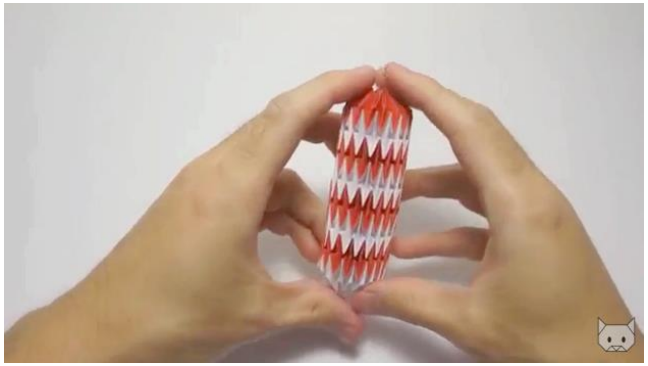


Origami of DNA Origami

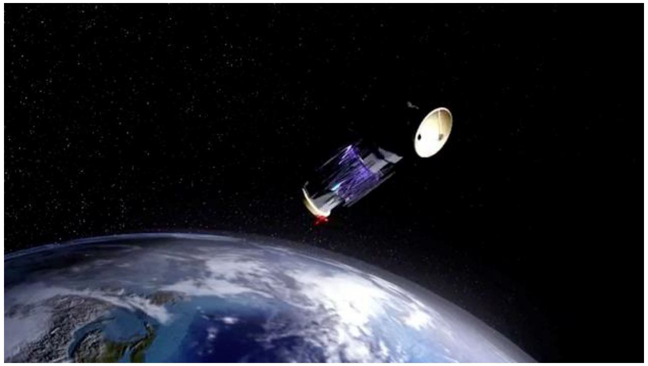
Design of DNA Origami Waterbomb

- **Paper origami art**



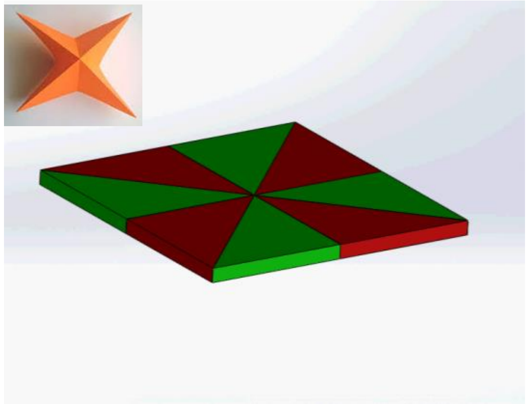


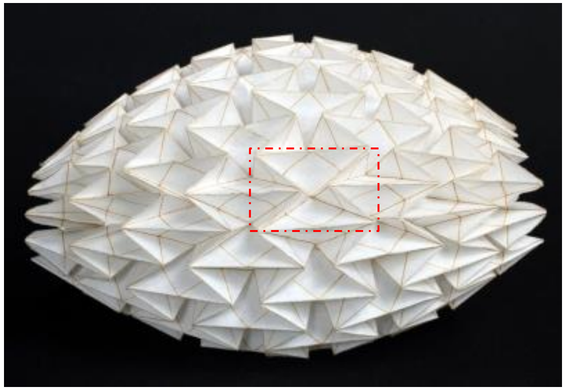
Yuri Shumakov, (video by Jo Nakashima)



Satellite concept design, Prof. Larry Howell BYU

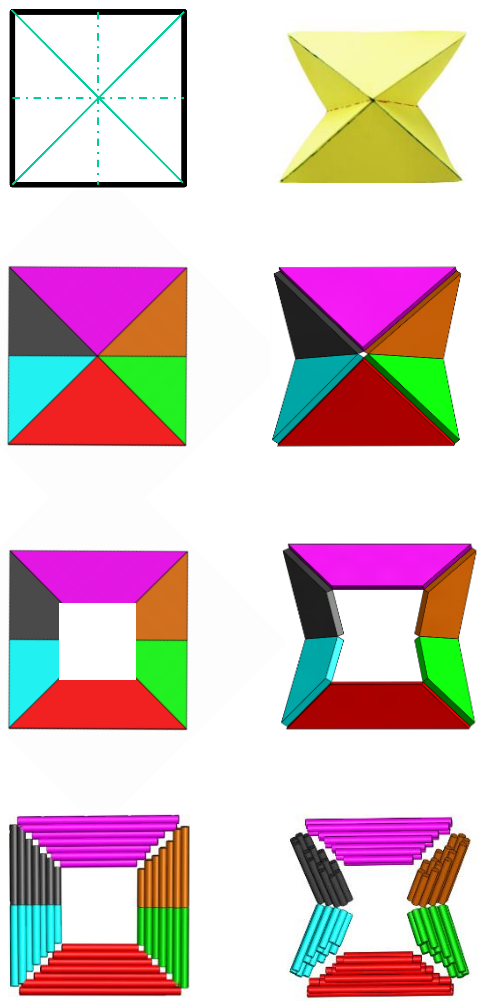
- **Waterbomb base**

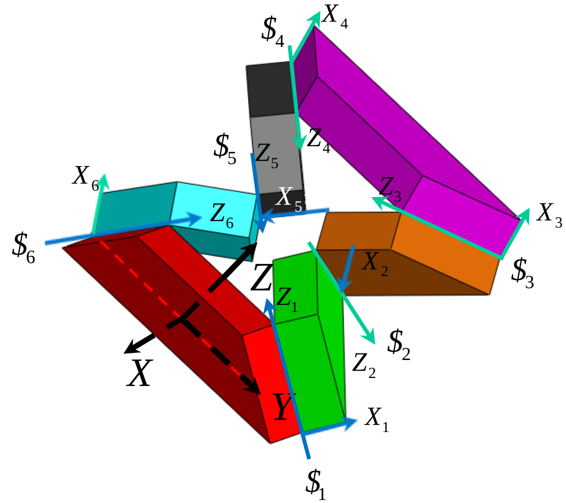


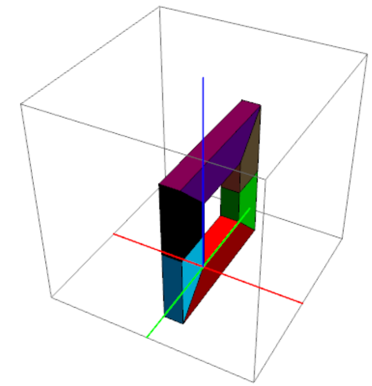


(https://bryantyee.wordpress.com/tag/waterbomb-base/)

Design of Thick Origami Waterbomb



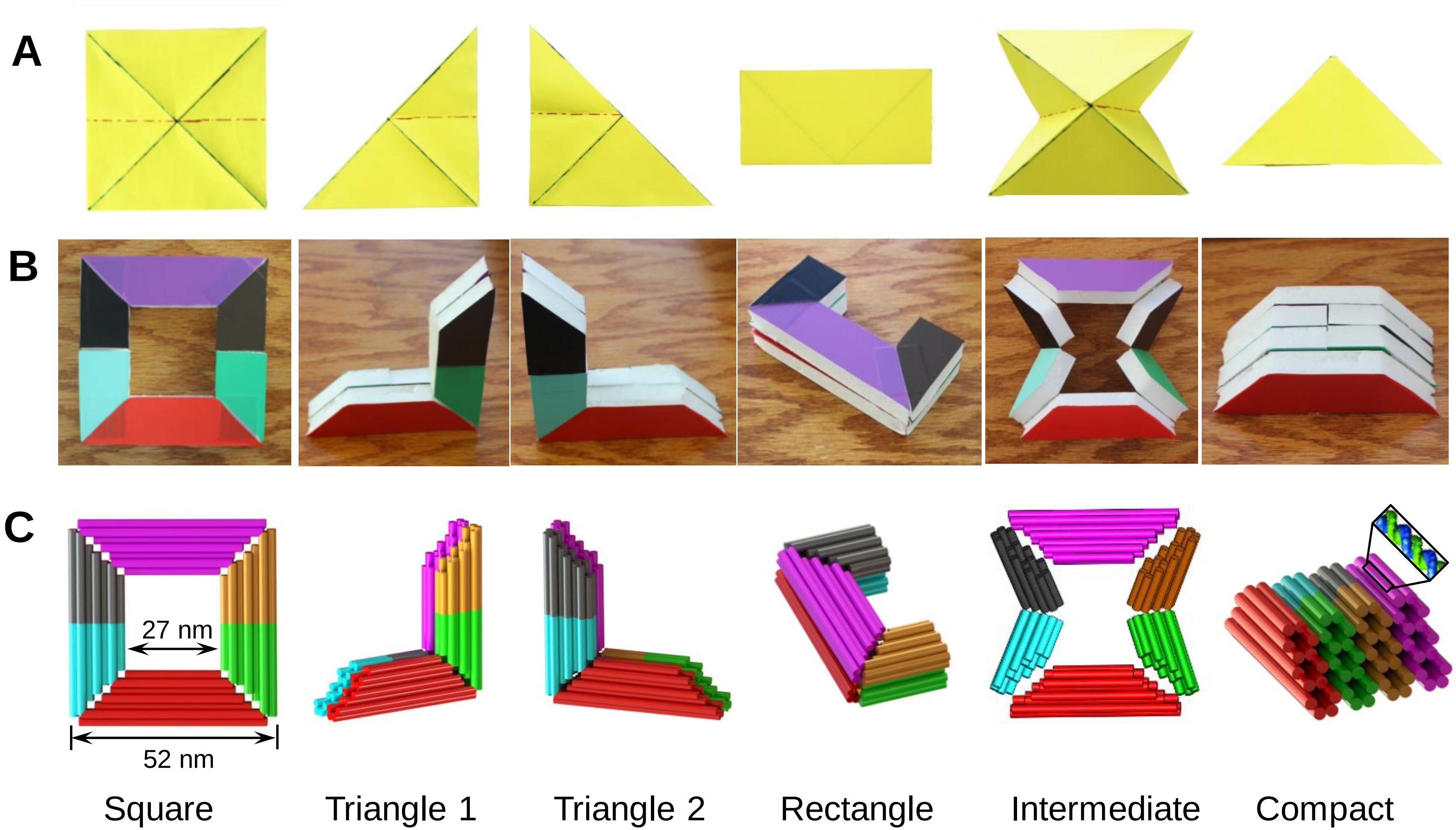




Joint i	$\alpha_{i\text{-}1}$	$a_{i\text{-}1}$	d_i	θ_i	$[R_i]$	$\mathbf{r}_{i\text{-}1}$	$\$_i$
1	$\pi/4$	0	0	θ_1	$R_X(\pi/4)R_Z(\theta_1)$	$(0,0,0)$	$\$_1=[Ad]_1\$_0$
2	$-3\pi/4$	t	0	θ_2	$R_X(-3\pi/4)R_Z(\theta_2)$	$(t,0,0)$	$\$_2=[Ad]_1[Ad]_2\$_0$
3	$-3\pi/4$	$-t$	0	θ_3	$R_X(-3\pi/4)R_Z(\theta_3)$	$(-t,0,0)$	$\$_3=[Ad]_1[Ad]_2[Ad]_3\$_0$
6	$-\pi/4$	0	0	θ_6	$R_X(-\pi/4)R_Z(\theta_6)$	$(0,0,0)$	$\$_6=[Ad]_6[Ad]_5[Ad]_4\$_0$
5	$3\pi/4$	t	0	θ_5	$R_X(3\pi/4)R_Z(\theta_5)$	$(t,0,0)$	$\$_5=[Ad]_6[Ad]_5\$_0$
4	$3\pi/4$	$-t$	0	θ_4	$R_X(3\pi/4)R_Z(\theta_4)$	$(-t,0,0)$	$\$_4=[Ad]_6\$_0$

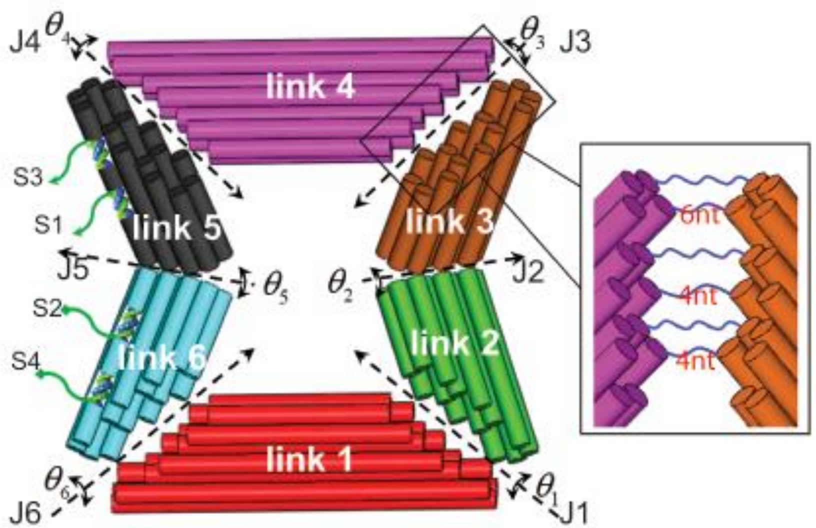
$$-1+\cos\theta_1^2+\cos\theta_2-\cos\theta_1^2\cos\theta_2+2\cos\theta_2\sin\theta_1^2-2\sqrt{2}\cos\theta_1\sin\theta_1\sin\theta_2=0$$

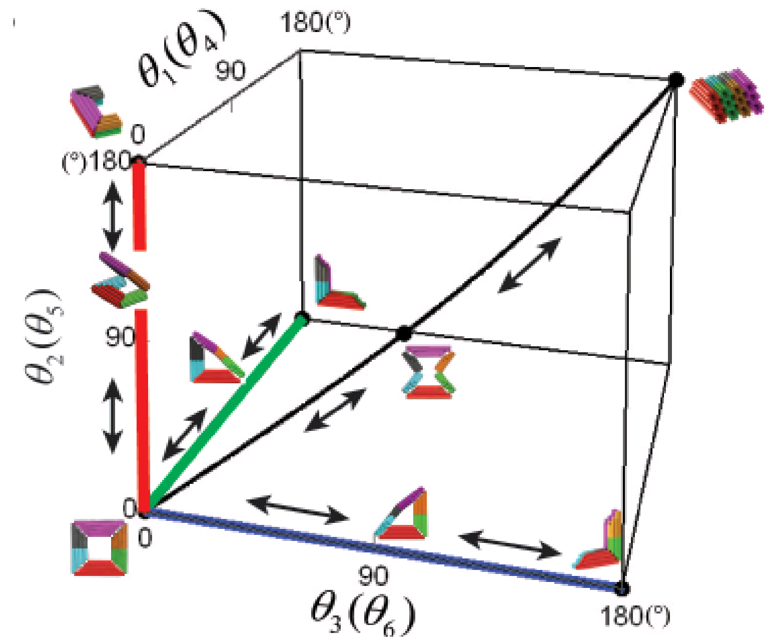
Different Configurations

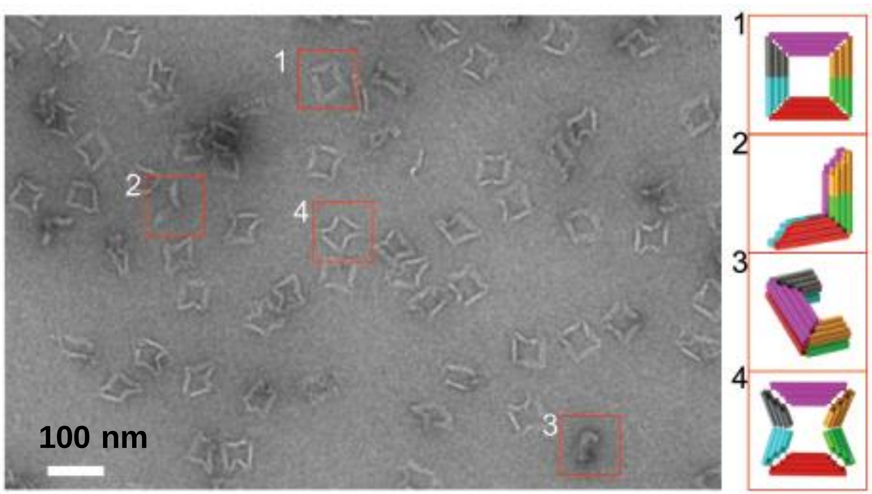


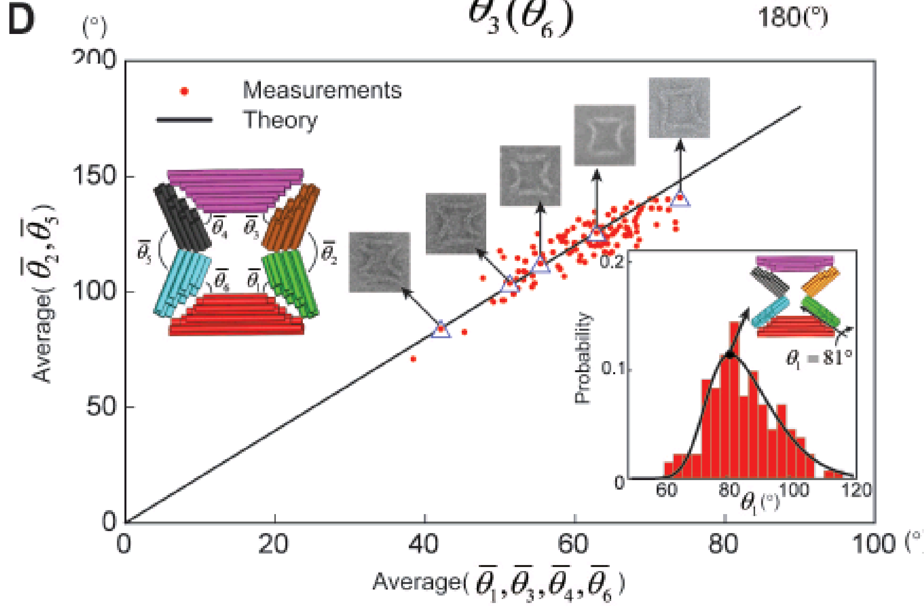
A. Single paper origami; **B**. thick panel origami; **C**. cylinder model.

Experimental Data and Analysis

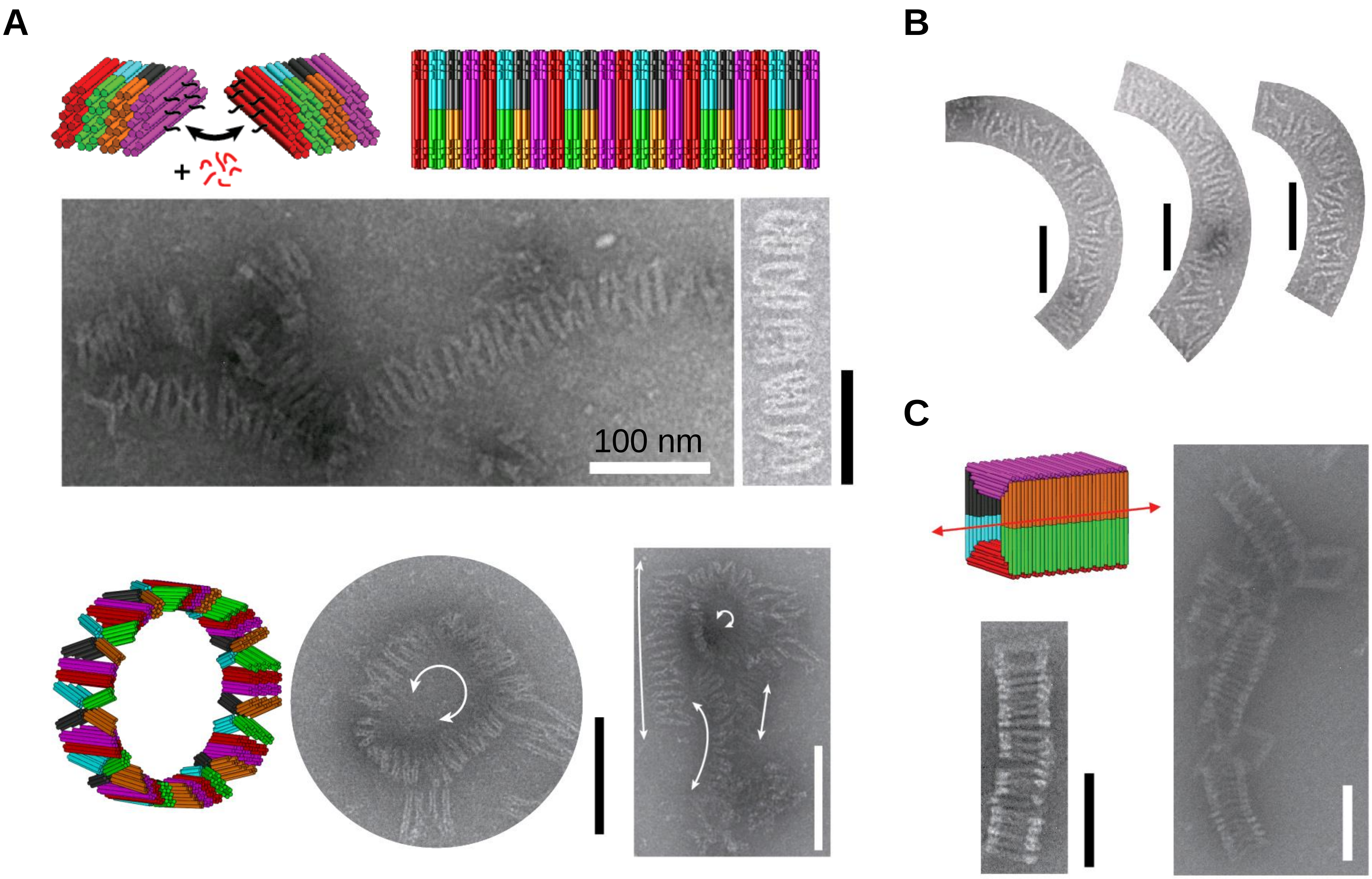




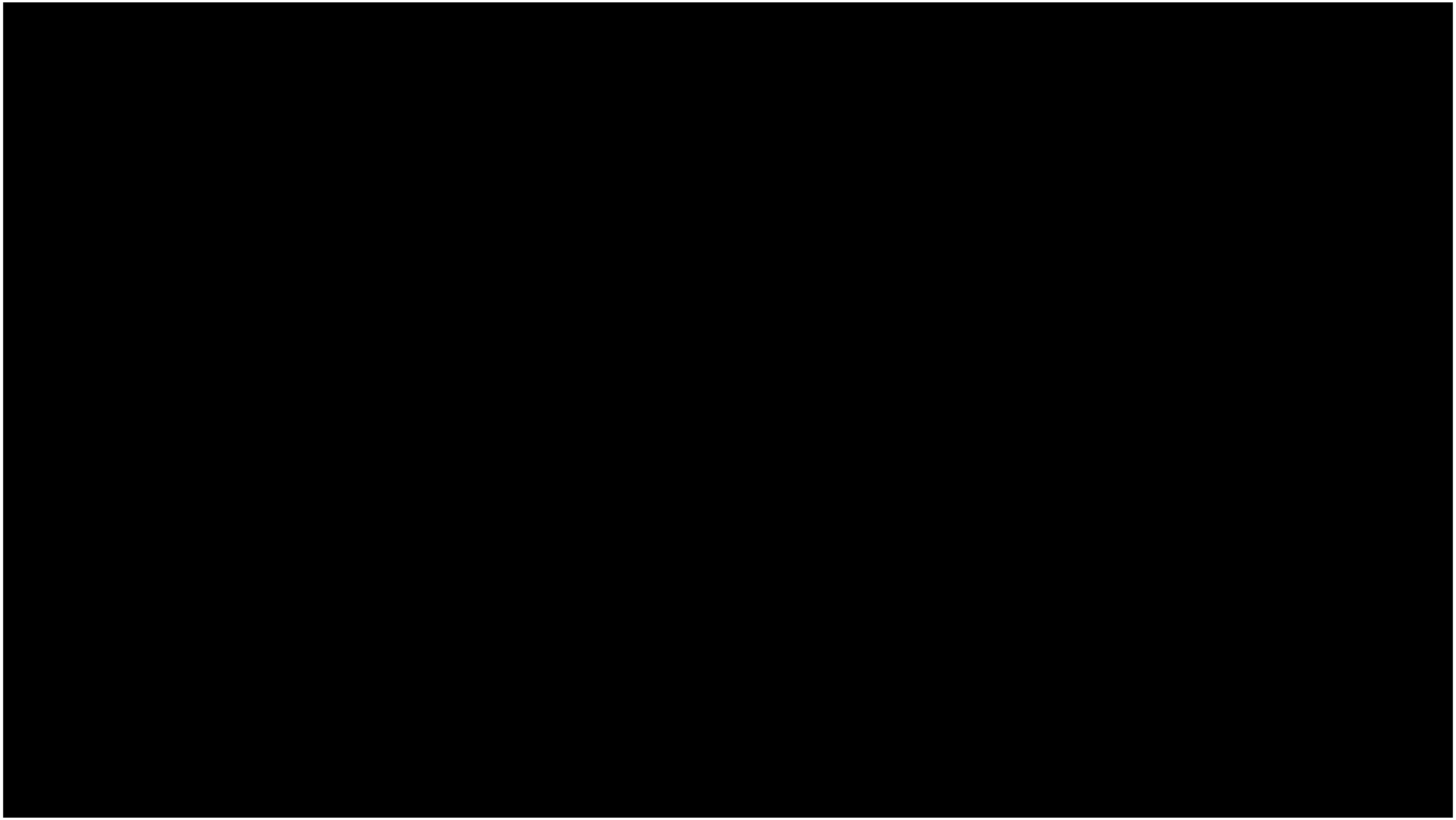




Polymerization of Multiple Units



Movie



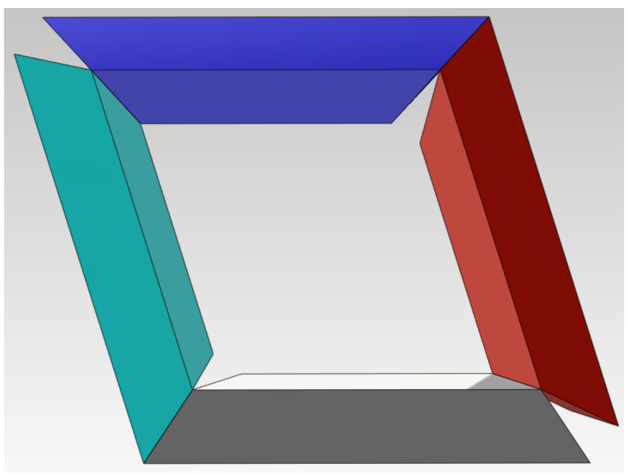
Configuration Analysis of DNA Origami Mechanisms via Projection Kinematics

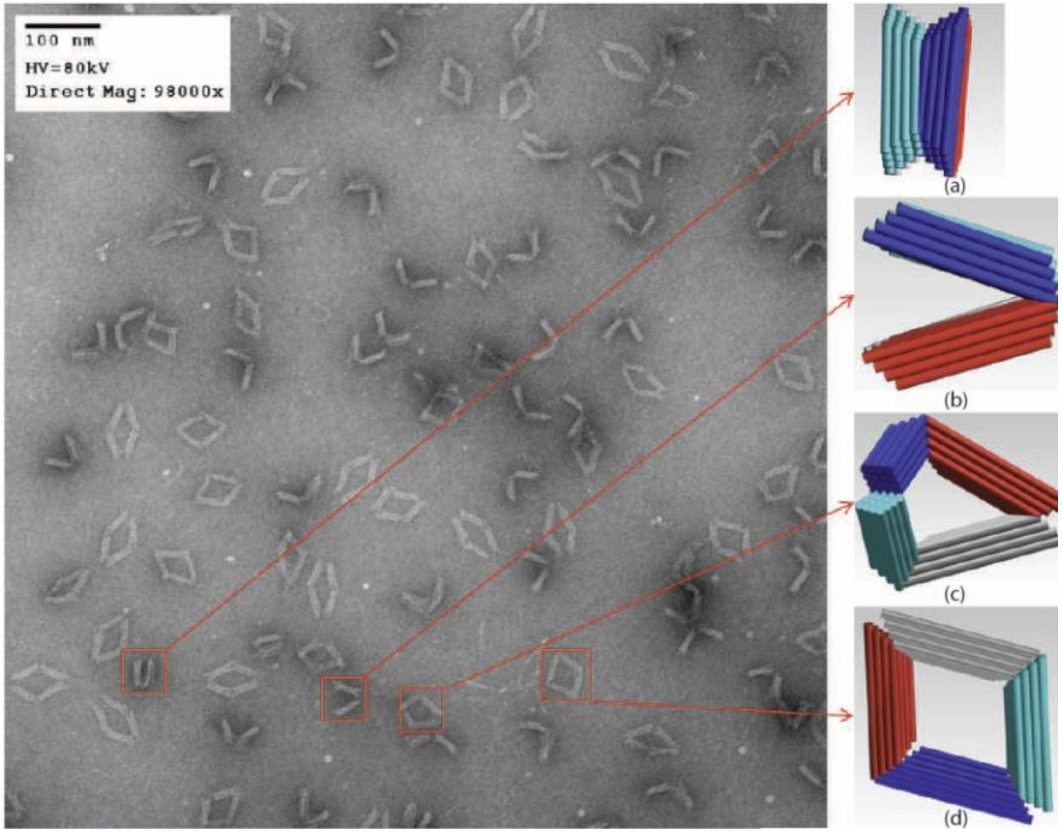
Zhou et al., Mechanism and Machine Theory, 2017

Zhou, Ph.D. Disseration, Ohio State University, 2017

Motivations of Projection Kinematics

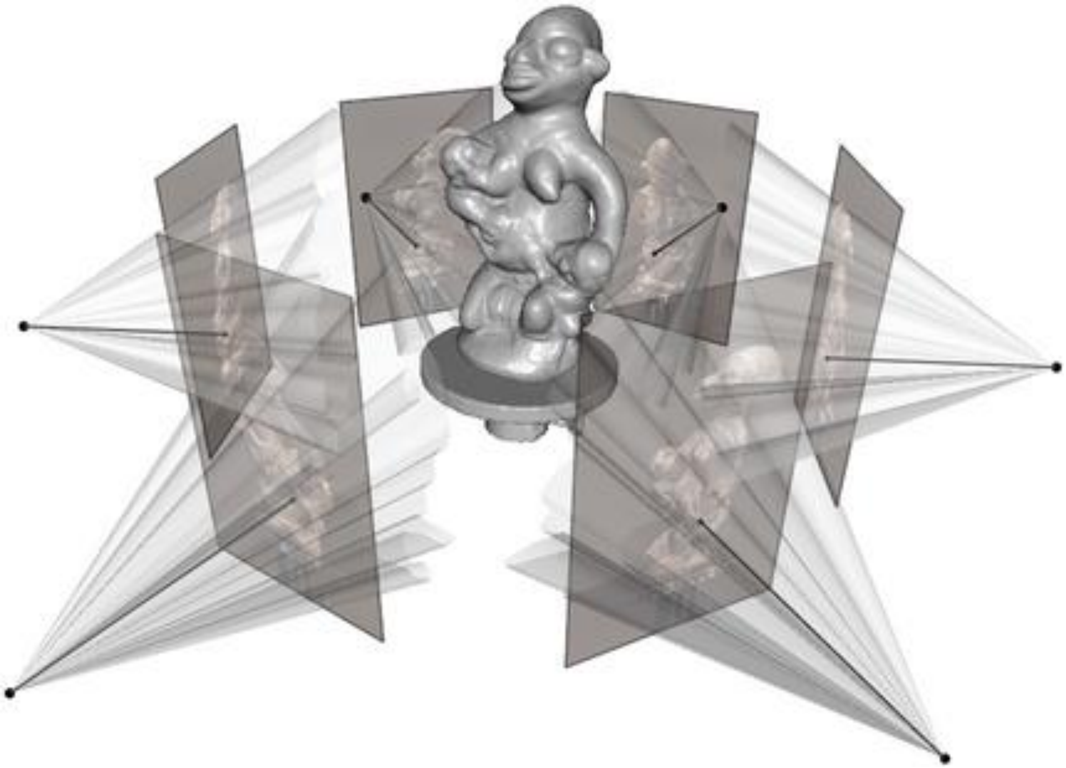
- **Configuration analysis: evaluate how well the prototype samples meet the original design goals**
- **Challenges for DOM: samples with mobility (every sample has a different configuration, only projected parameters are measurable.**





Reconstruction of 3D Objects

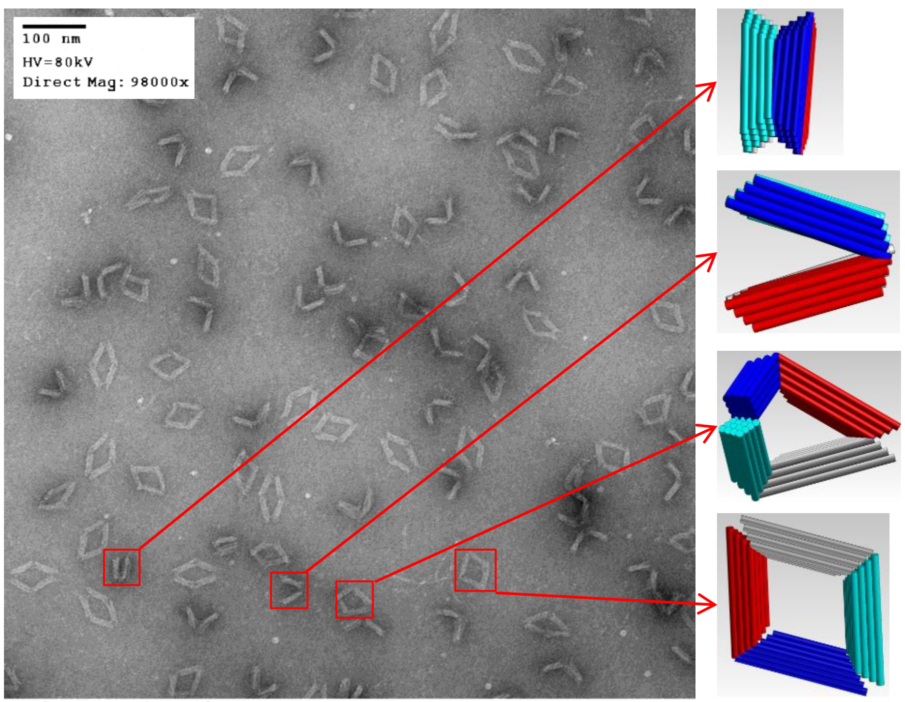
- **In computer vision, reconstruction of 3D Objects from 2D images is limited to static (no mobility) objects and requires multiple images from different view angles.**

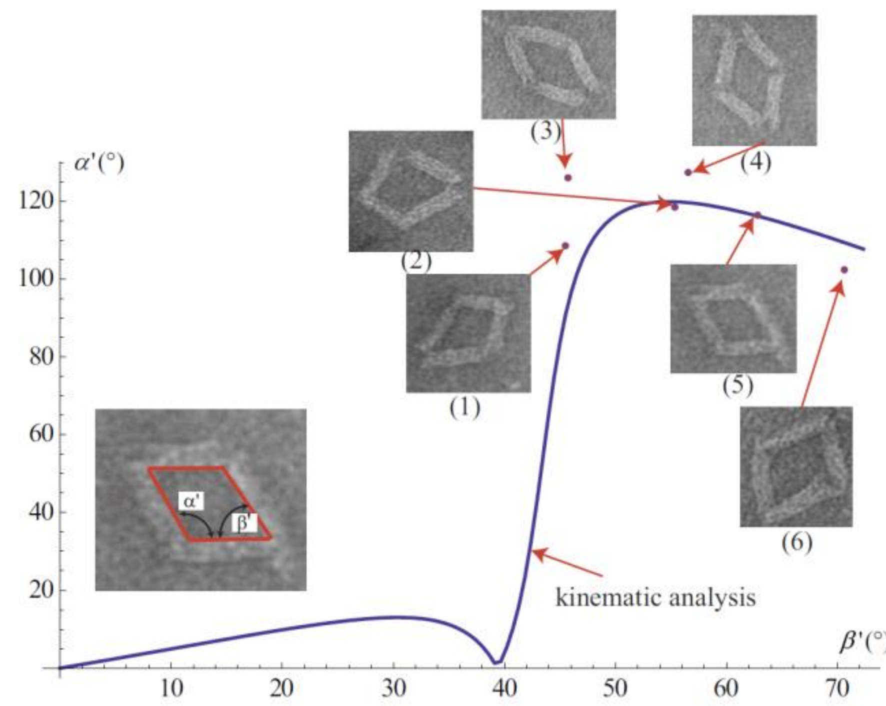


Configuration Analysis of DOM

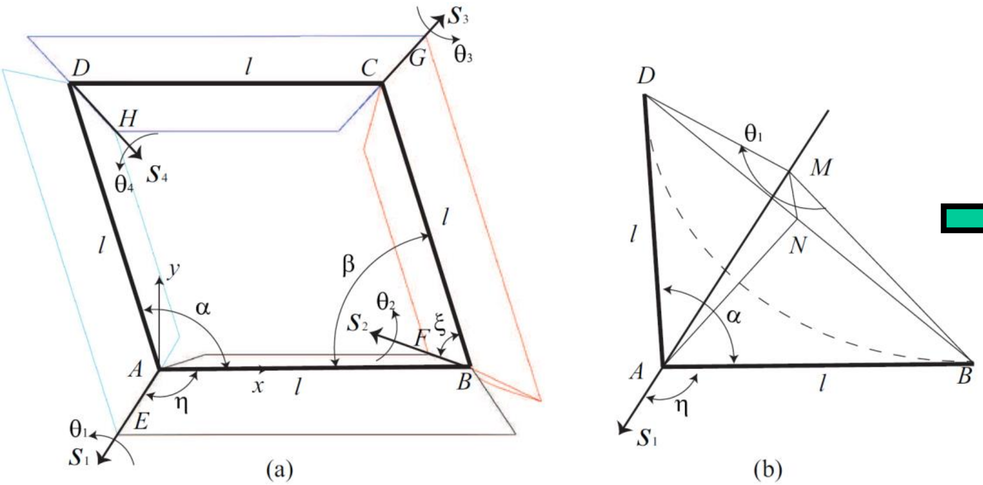
- Evaluate how well the prototype samples meet the original design goals
- Measure key kinematic parameters and compare against kinematic constraint equations

$$(\mathbf{D}-\mathbf{C})\cdot(\mathbf{D}-\mathbf{C})-l^2=0 \quad \beta'=\beta, \quad \alpha'=\cos^{-1}\left(\frac{\mathbf{D}'\cdot\mathbf{B}}{l^2}\right)$$



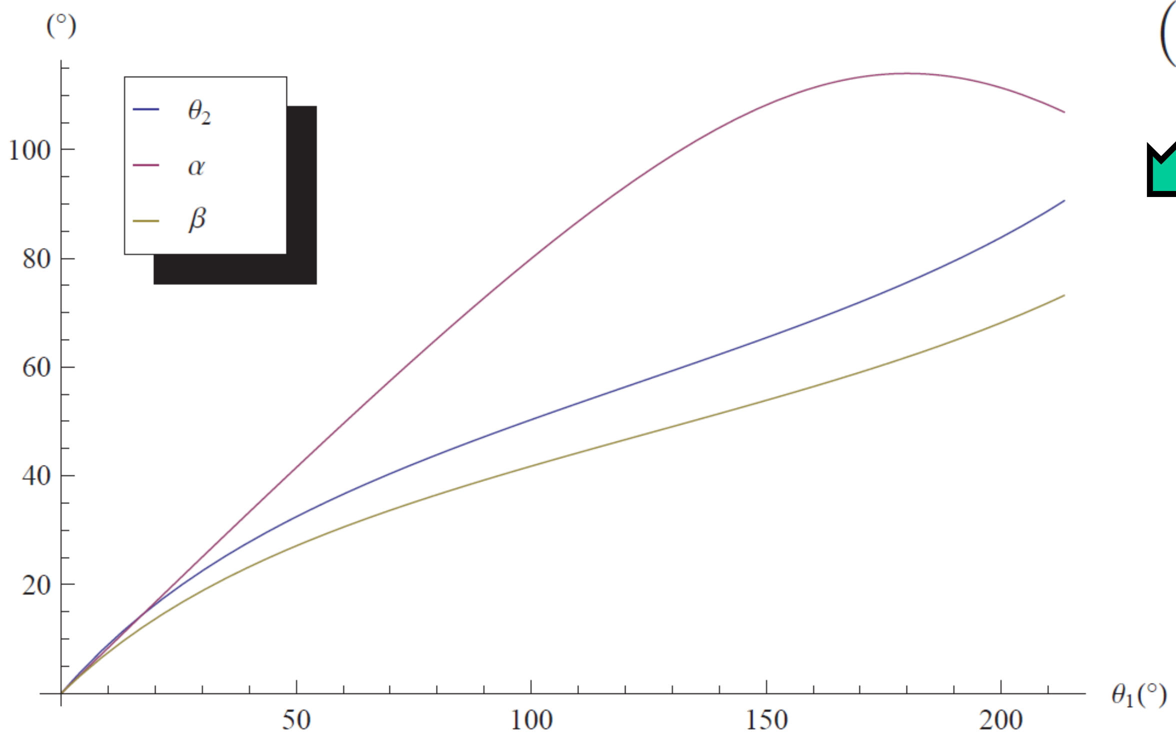


Kinematics of The Bennett Linkage



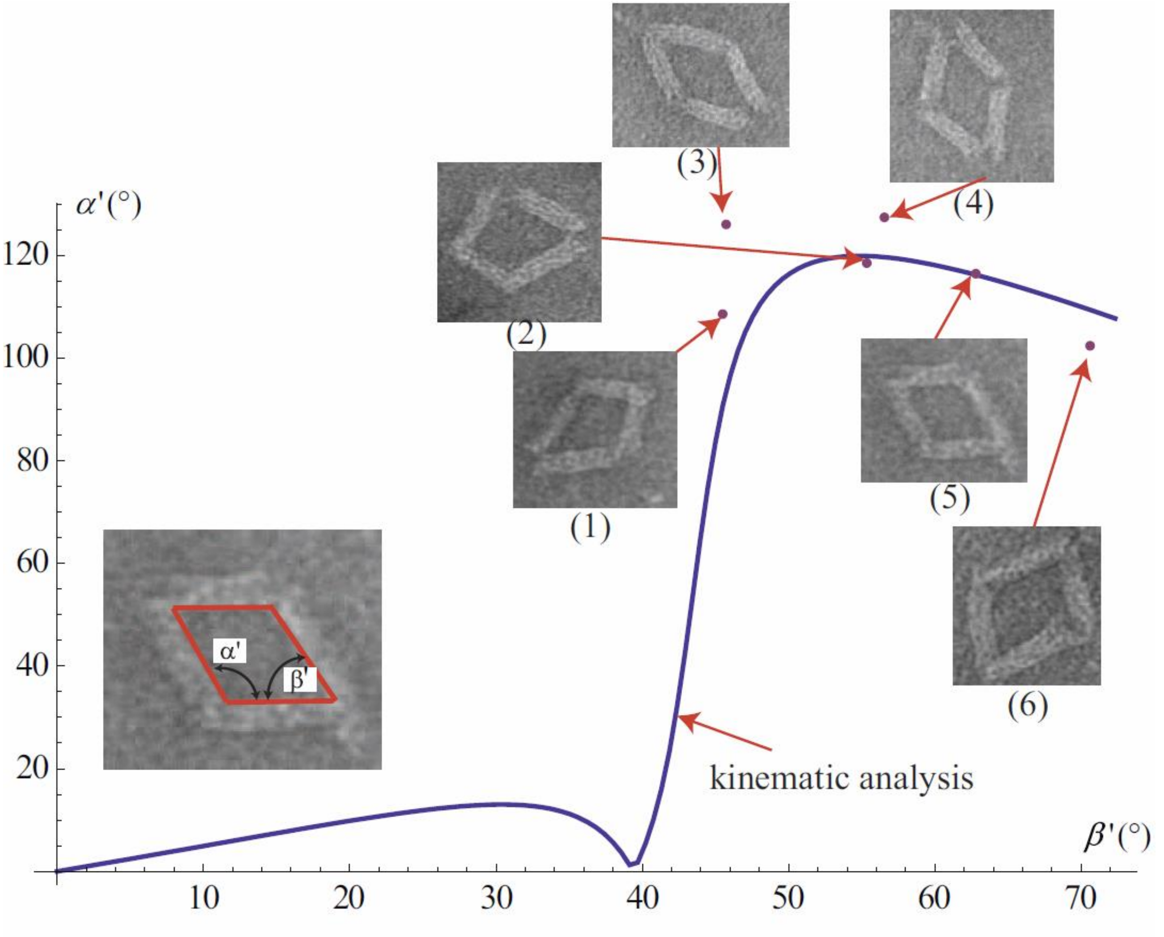
$$\mathbf{D} = \left[e^{\theta_1 \mathbf{S}_1}\right] \begin{Bmatrix} l \\ 0 \\ 0 \end{Bmatrix}, \quad \mathbf{C} = \mathbf{B} + \left[e^{\theta_2 \mathbf{S}_2}\right] \begin{Bmatrix} -l \\ 0 \\ 0 \end{Bmatrix}$$

$$(\mathbf{D} - \mathbf{C}) \cdot (\mathbf{D} - \mathbf{C}) - l^2 = 0$$



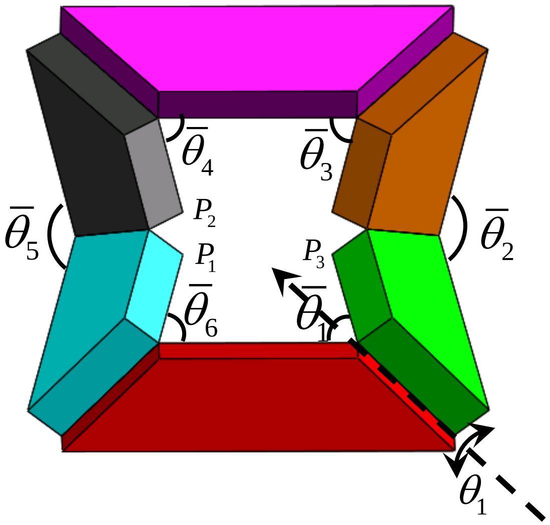
Projection Kinematics

$$\beta' = \beta, \quad \alpha' = \cos^{-1}\left(\frac{\mathbf{D}' \cdot \mathbf{B}}{l^2}\right)$$

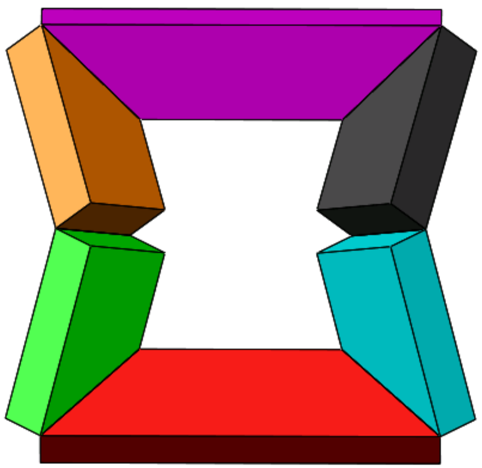


Projection Kinematic Analysis of Waterbomb Base

Top view

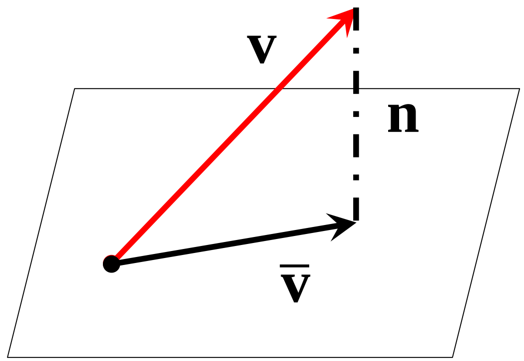


Bottom view



$$-1+\cos\theta_1^2+\cos\theta_2-\cos\theta_1^2\cos\theta_2+2\cos\theta_2\sin\theta_1^2-2\sqrt{2}\cos\theta_1\sin\theta_1\sin\theta_2=0$$

Projection from 3D to a 2D Plane



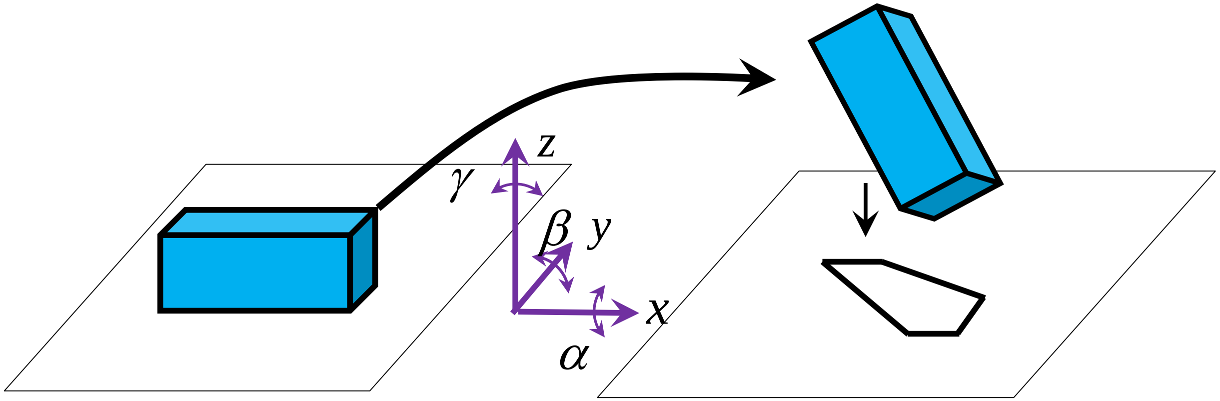
$$\bar{\mathbf{v}} = \mathbf{v} - (\mathbf{v} \cdot \mathbf{n})\mathbf{n} = [p]\mathbf{v}$$

v: Arbitrary vector;
$\bar{\mathbf{v}}$: Projection vector;
n: Projection direction.

n={0,0,1} $[p] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Projection of 3D Objects

- **Relationship between the projection configuration and the true configuration in 3D space can be obtained from the kinematics analysis and transformations.**



$$[X(\alpha)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \qquad [Y(\beta)] = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$[R] = [Y(\beta)][X(\alpha)] = \begin{bmatrix} \cos\beta & \sin\alpha\sin\beta & \cos\alpha\sin\beta \\ 0 & \cos\alpha & -\sin\alpha \\ -\sin\beta & \sin\alpha\cos\beta & \cos\alpha\cos\beta \end{bmatrix}$$

Projection of Kinematic Pairs

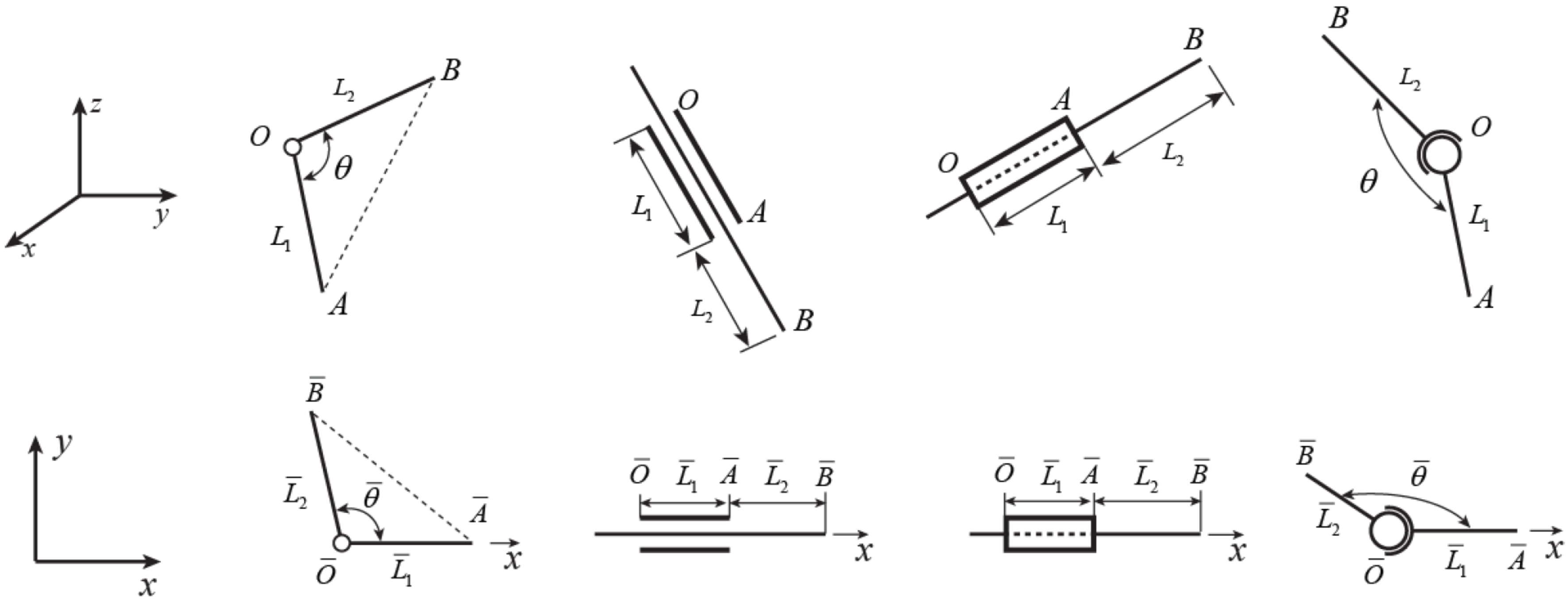
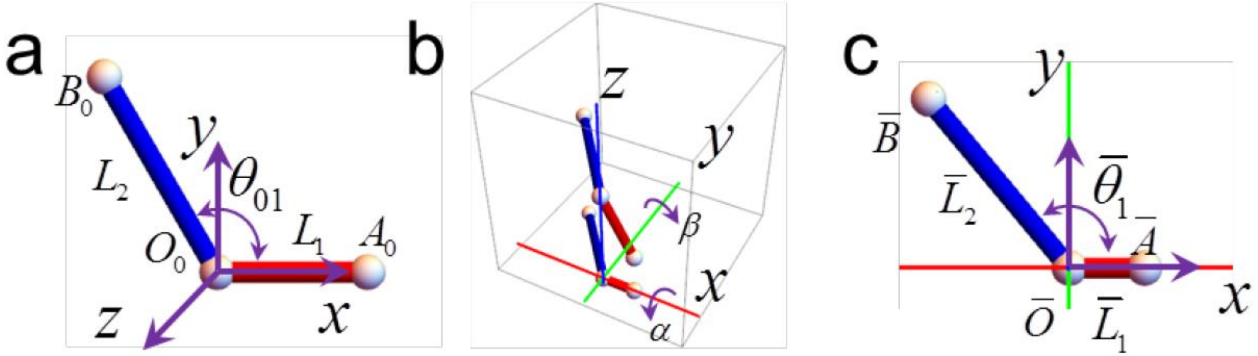


Fig. 1. Kinematic parameters of revolute, prismatic, cylindrical and spherical joints (from left to right) are 3D (top) in nature. They are projected to a *x*-*y* plane (bottom).

Projection Kinematics of R-Joint

- **Projection Kinematic Equations**



$$\begin{cases} L_1 \cos\beta = \bar{L}_1 \\ L_2(\cos\theta_{01}\cos\beta + \sin\theta_{01}\sin\alpha\sin\beta) = \bar{L}_2\cos\bar{\theta}_1 \\ L_2\sin\theta_{01}\cos\alpha = \bar{L}_2\sin\bar{\theta}_1 \end{cases}$$

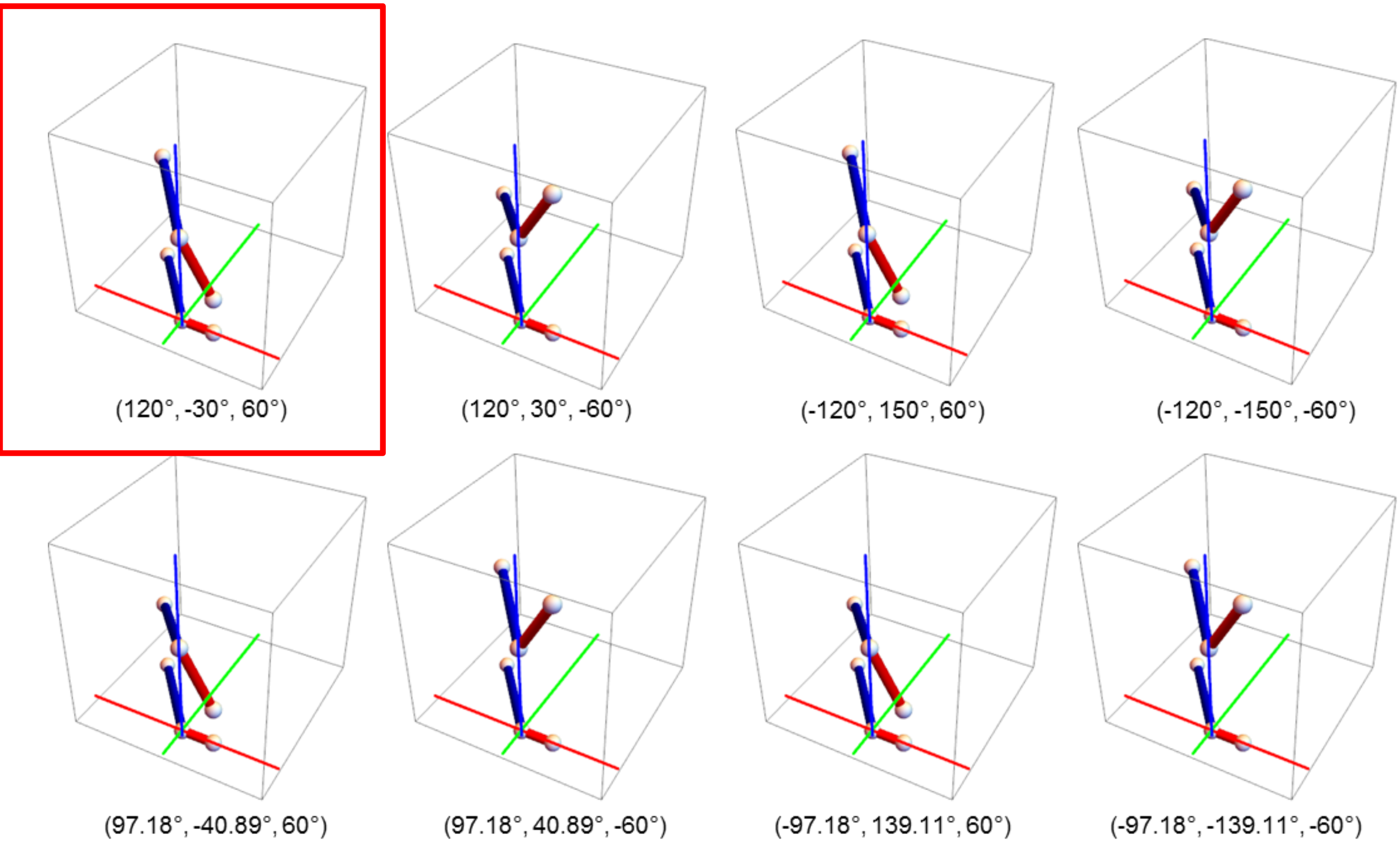
$$\boxed{\begin{array}{l} (\theta_{01}, \alpha, \beta), \\ (\theta_{01}, -\alpha, -\beta), \\ (-\theta_{01}, \pi-\alpha, \beta), \\ (-\theta_{01}, \alpha-\pi, \beta) \end{array}}$$

Points in the local reference frame	Points after rotation	Projected points	Measured points from a 2D image
$\mathbf{A}_0 = (L_1, 0, 0)^T$	$\mathbf{A} = [R]\mathbf{A}_0$	$\bar{\mathbf{A}} = [P]\mathbf{A}$	$\bar{\mathbf{A}} = (\bar{L}_1, 0, 0)^T$
$\mathbf{B}_0 = (L_2\cos\theta_{01}, L_2\sin\theta_{01}, 0)^T$	$\mathbf{B} = [R]\mathbf{B}_0$	$\bar{\mathbf{B}} = [P]\mathbf{B}$	$\bar{\mathbf{B}} = (\bar{L}_2\cos\bar{\theta}_1, \bar{L}_2\sin\bar{\theta}_1, 0)^T$

Numerical Example

- **Eight possible configurations**

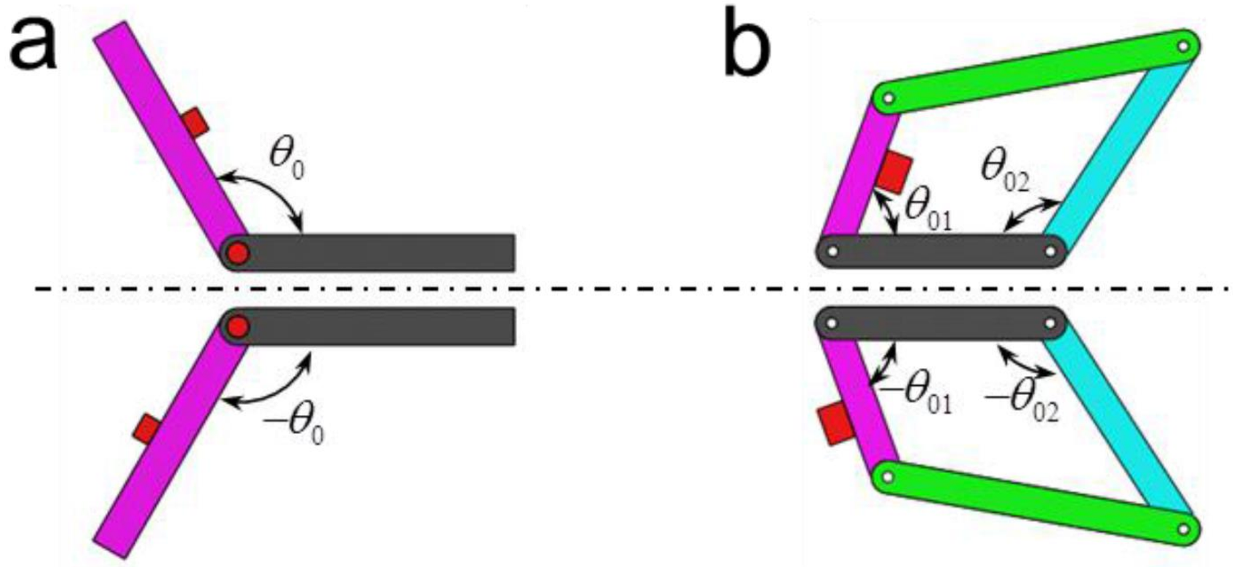
Initial configuration	L_1=100, L_2=150, θ_{01} =120°
Rotation angles	α = -30°, β = 60°
Projected configuration	$\bar{L}_1$=50, $\bar{L}_2$=195.256, $\bar{\theta}_1$=129.809°



$(\theta_{01}, \alpha, \beta)$

$$(\theta_{01}, \alpha, \beta),$$
$$(\theta_{01}, -\alpha, -\beta),$$
$$(-\theta_{01}, \pi-\alpha, \beta),$$
$$(-\theta_{01}, \alpha-\pi, \beta)$$

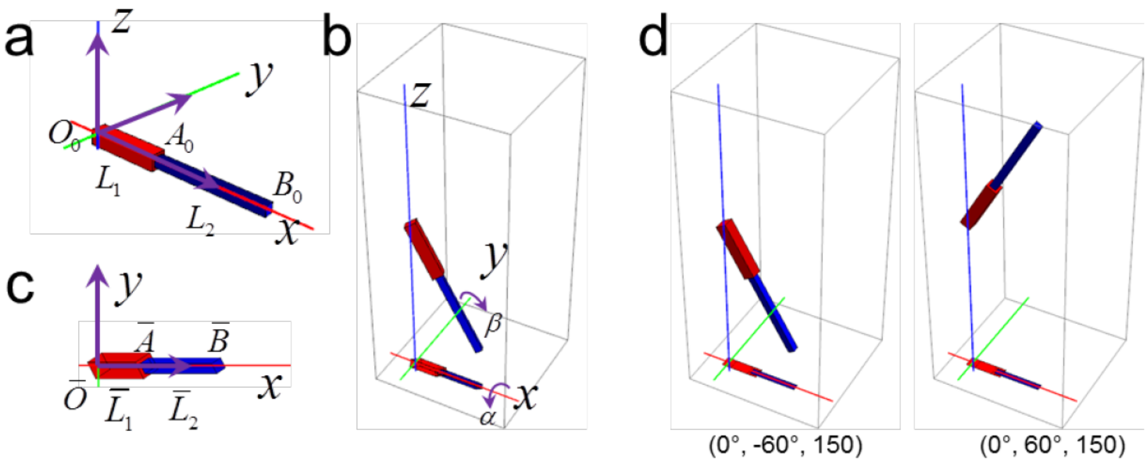
Symmetric Configurations



Symmetrical (mirrored) configurations distinguished by a designed feature (red box on the purple link). (a) Positive and negative revolute joint angles. (b) Symmetrical configurations of planer four-bar linkage.

Projection Kinematics of P/C-Joints

- **Prismatic Joints (two configurations)**



Projection kinematic equations

$$\begin{cases} L_1 \cos\beta = \overline{L}_1 \\ (L_1 + L_2)\cos\beta = \overline{L}_1 + \overline{L}_2 \end{cases}$$

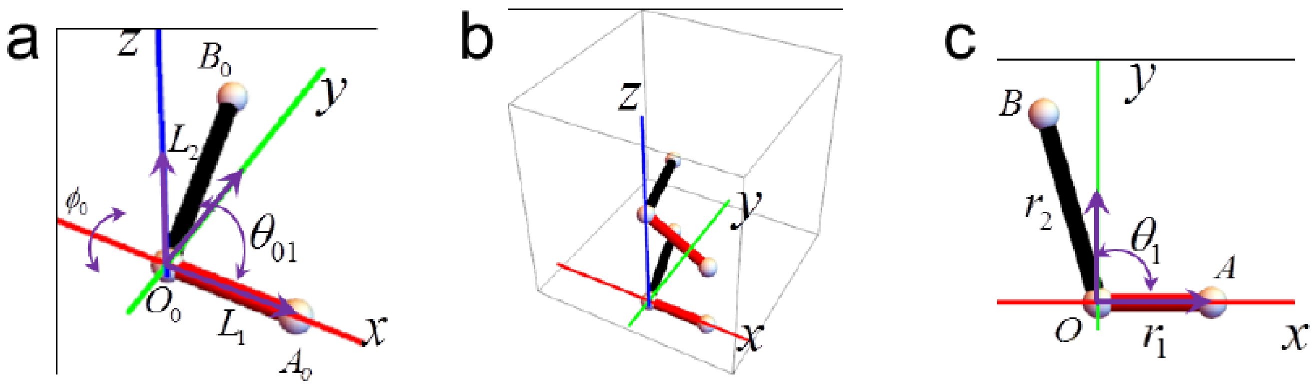
Numerical example

Initial configuration	L_1=100, L_2=150
Rotation angles	α = -30°, β = 60°
Projected configuration	$\overline{L}_1$=50, $\overline{L}_2$=75

- **Cylindrical Joints (same as prismatic joints)**

Projection Kinematics of S-Joints

- **Projection kinematic equations**



Treat α+ϕ_0 as one variable

$$\begin{cases} L_1 \cos\beta = \bar{L}_1 \\ L_2(\cos\theta_{01}\cos\beta + \sin\theta_{01}\cos(\alpha+\phi_0)\sin\beta) = \bar{L}_2\cos\bar{\theta}_1 \\ L_2 \sin\theta_{01}\sin(\alpha+\phi_0) = \bar{L}_2\sin\bar{\theta}_1 \end{cases}$$

Points in the local reference frame	Points after rotation	Projected points	Measured points from a 2D image
$\mathbf{A}_0=(L_1,0,0)^T$	$\mathbf{A}=[R]\mathbf{A}_0$	$\bar{\mathbf{A}}=[P]\mathbf{A}$	$\bar{\mathbf{A}}=(\bar{L}_1,0,0)^T$
$\mathbf{B}_0=[X(\phi_0)]\mathbf{B}_{00}$	$\mathbf{B}=[R]\mathbf{B}_0$ $=[Y(\beta)][X(\alpha+\phi_0)]\mathbf{B}_{00}$	$\bar{\mathbf{B}}=[P]\mathbf{B}$	$\bar{\mathbf{B}}=(\bar{L}_2\cos\bar{\theta}_1,\bar{L}_2\sin\bar{\theta}_1,0)^T$

Here $\mathbf{B}_{00}=(L_2\cos\theta_{01},L_2\sin\theta_{01},0)^T$.

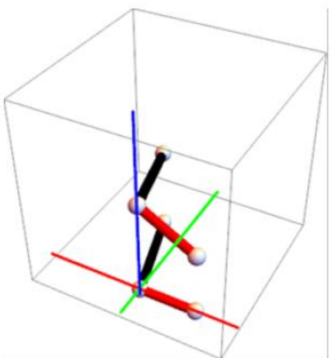
Numerical Example

- **Eight possible configurations**

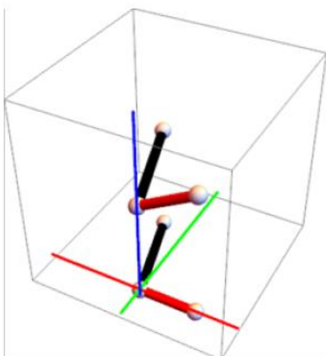
Treat $\alpha+\phi_0$ as one variable

Initial configuration	L_1=100, L_2=150; θ_{01}=100°, ϕ_0= 15°
Rotation angles	α = -30°, β=30°
Projected configuration	$\bar{L}_1$=86.60, $\bar{L}_2$=148.65, $\bar{\theta}_1$=106.28°

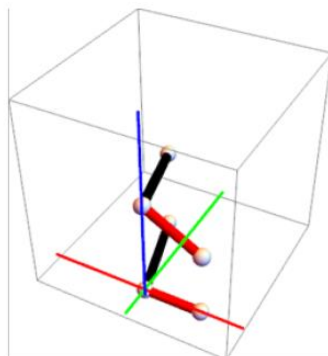
$\theta_{01},\beta,\alpha+\phi_0$



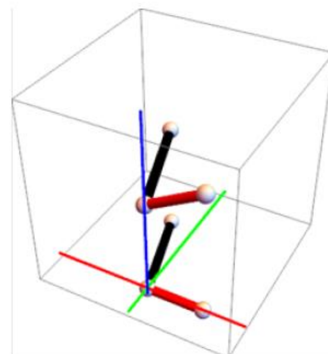
(100°, 30°, -15°)



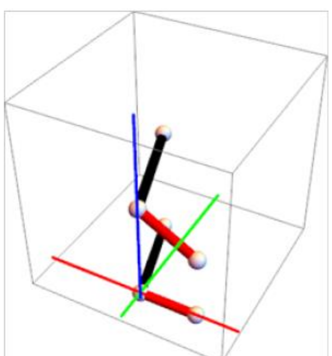
(100°, -30°, 15°)



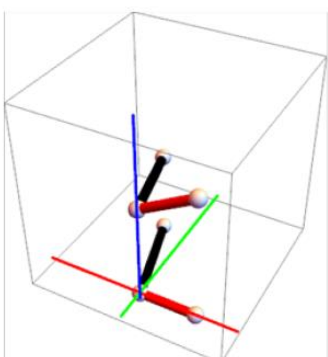
(-100°, -30°, 165°)



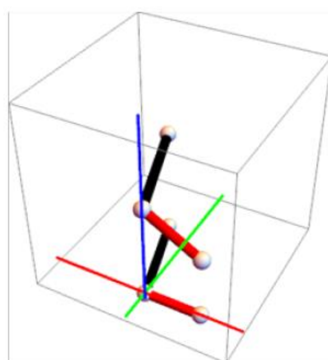
(-100°, -30°, -165°)



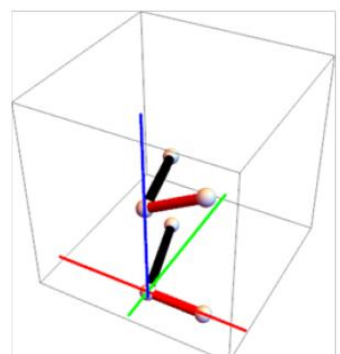
(107.91°, 30°, -1.38°)



(107.91°, -30°, 1.38°)



(-107.91°, 30°, 178.62°)



(-107.91°, -30°, -178.62°)

Projection Kinematics of Planar 4-Bar

- **Projection kinematic equations**

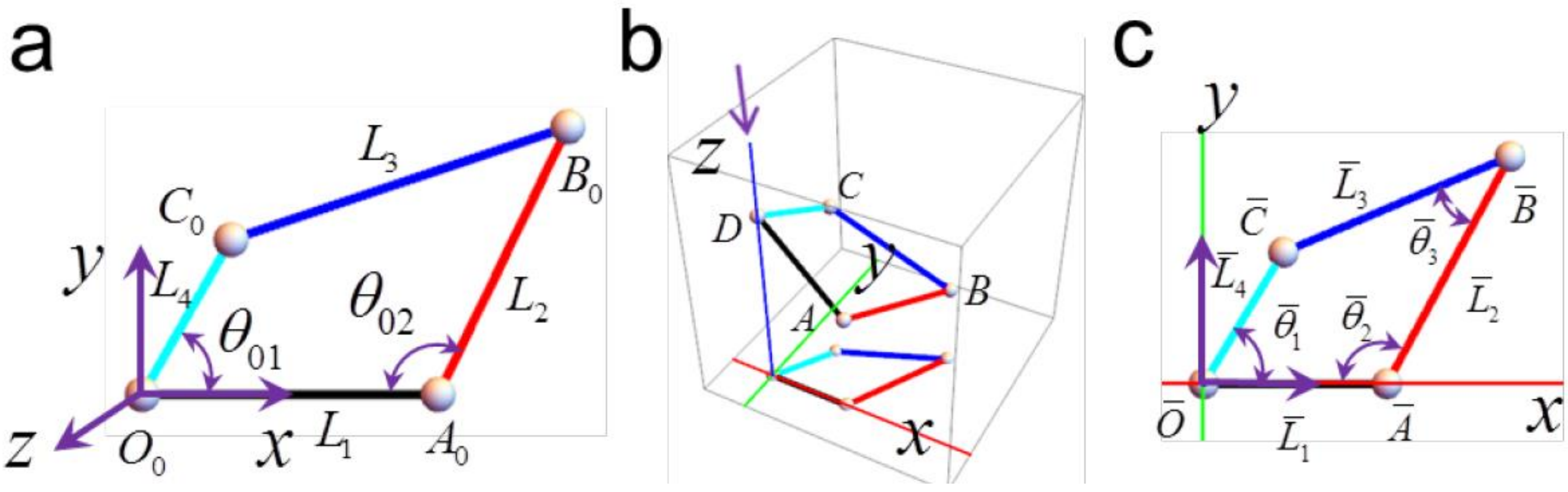
$$L_1 \cos\beta = \bar{L}_1 \tag{7}$$

$$L_1 \cos\beta - L_2 \cos\beta \cos\theta_{02} + L_2 \sin\alpha \sin\beta \sin\theta_{02} = \bar{L}_1 + \bar{L}_2 \cos(\pi - \bar{\theta}_2) \tag{8}$$

$$L_2 \cos\alpha \sin\theta_{02} = \bar{L}_2 \sin(\pi - \bar{\theta}_2) \tag{9}$$

$$L_4 \cos\beta \cos\theta_{01} + L_4 \sin\alpha \sin\beta \sin\theta_{01} = \bar{L}_4 \cos\bar{\theta}_1 \tag{10}$$

$$L_4 \cos\alpha \sin\theta_{01} = \bar{L}_4 \sin\bar{\theta}_1 \tag{11}$$



Projection Kinematics of Planar 4-Bar

- ## Projection kinematic equations

$$L_1 \cos\beta = \overline{L}_1 \tag{7}$$

$$L_1 \cos\beta - L_2 \cos\beta \cos\theta_{02} + L_2 \sin\alpha \sin\beta \sin\theta_{02} = \overline{L}_1 + \overline{L}_2 \cos(\pi - \overline{\theta}_2) \tag{8}$$

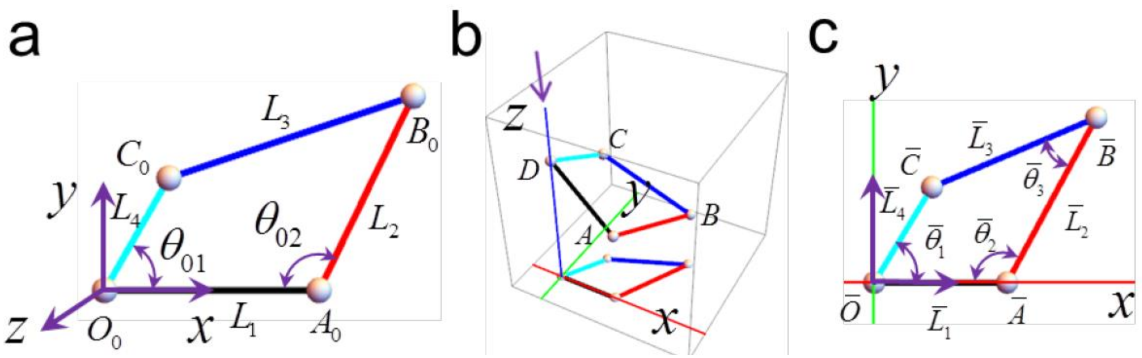
$$L_2 \cos\alpha \sin\theta_{02} = \overline{L}_2 \sin(\pi - \overline{\theta}_2) \tag{9}$$

$$L_4 \cos\beta \cos\theta_{01} + L_4 \sin\alpha \sin\beta \sin\theta_{01} = \overline{L}_4 \cos\overline{\theta}_1 \tag{10}$$

$$L_4 \cos\alpha \sin\theta_{01} = \overline{L}_4 \sin\overline{\theta}_1 \tag{11}$$

- ## Vector Loop Closure Equation

$$L_1^{\,2} + L_2^{\,2} + L_4^{\,2} - L_3^{\,2} - 2L_1L_4 \cos\theta_{01} - 2L_1L_2 \cos\theta_{02} + 2L_2L_4 \cos(\theta_{01} + \theta_{02}) = 0 \tag{12}$$

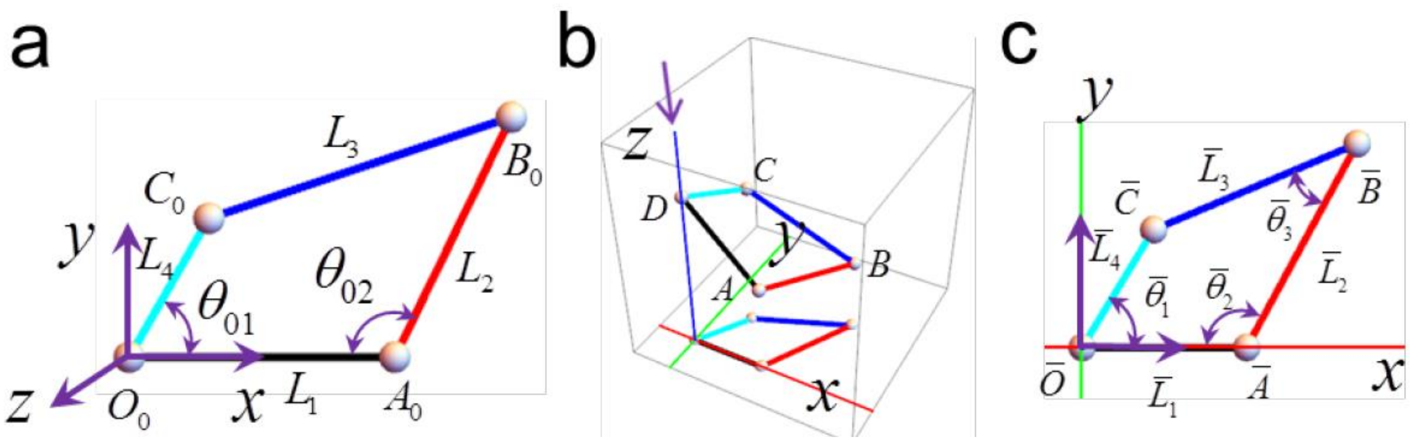


Measurement Cases of 4-bar

- **Projection kinematic equations**

Table 5.7: Equations and solutions of four kinds of projection kinematic analysis for the four-bar linkage

Case	Measured geometric parameters	Measured parameters	Representative Equations	# of solutions
1	Two links and the joint between them	$\bar{L}_1$, $\bar{L}_4$, $\bar{\theta}_1$	(7, 10, 11, 12)	2
2	Two adjacent joints and the link between them	$\bar{\theta}_1$, $\bar{L}_1$, $\bar{\theta}_2$	(7, 12, 15, 16)	4
3	Three joints	$\bar{\theta}_1$, $\bar{\theta}_2$, $\bar{\theta}_3$	(12, 20, 21, 22)	4
4	Three links	$\bar{L}_1$, $\bar{L}_2$, $\bar{L}_4$	(7, 12, 24, 25)	8



Numerical Examples of Possible Configurations of Planar 4-Bar

Initial configuration	L_1=5, L_2=5, L_3=6, L_4=3, θ_{01}=60°, θ_{02}=116.011°; α = 15°, β=45°
Projected configuration	$\bar{L}_1$=3.54, $\bar{L}_2$=4.95, $\bar{L}_3$=4.74, $\bar{L}_4$=2.94; $\bar{\theta}_1$=58.53°, $\bar{\theta}_2$=118.67°, $\bar{\theta}_3$=38.61°;

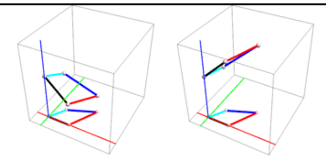
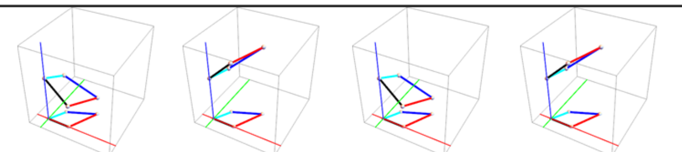
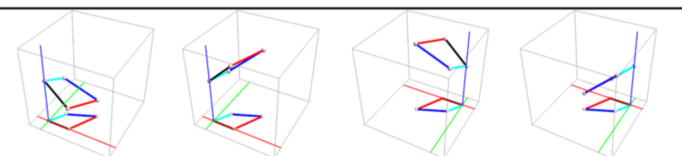
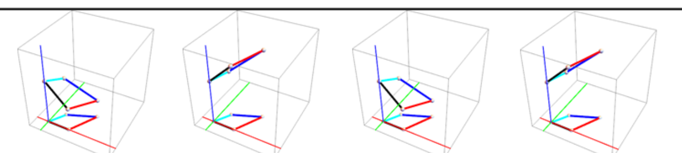
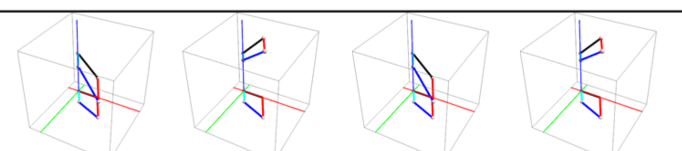
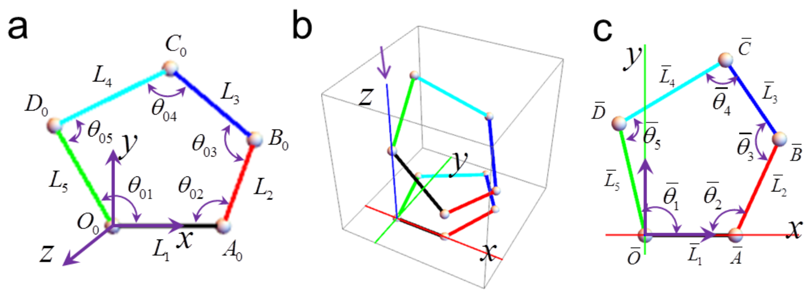
Case	Solutions
1	 (60°, 116.011°, 15°, 45°) (60°, 116.011°, -15°, -45°)
2	 (60°, 116.011°, 15°, 45°) (60°, 116.011°, -15°, -45°) (-60°, -116.011°, -165°, 45°) (-60°, -116.011°, 165°, -45°)
3	 (60°, 116.011°, 15°, 45°) (60°, 116.011°, -15°, -45°) (60°, 116.011°, 165°, -135°) (60°, 116.011°, -165°, 135°)
4	 (60°, 116.011°, 15°, 45°) (60°, 116.011°, -15°, -45°) (-60°, -116.011°, -165°, 45°) (-60°, -116.011°, 165°, -45°)
	 (60°, 116.011°, 165°, 45°) (60°, 116.011°, -165°, -45°) (-60°, -116.011°, -15°, 45°) (-60°, -116.011°, 15°, -45°)

Table 5.7: Equations and solutions of four kinds of projection kinematic analysis for the four-bar linkage

Case	Measured geometric parameters	Measured parameters	Representative Equations	# of solutions
1	Two links and the joint between them	$\bar{L}_1$, $\bar{L}_4$, $\bar{\theta}_1$	(7, 10, 11, 12)	2
2	Two adjacent joints and the link between them	$\bar{\theta}_1$, $\bar{L}_1$, $\bar{\theta}_2$	(7, 12, 15, 16)	4
3	Three joints	$\bar{\theta}_1$, $\bar{\theta}_2$, $\bar{\theta}_3$	(12, 20, 21, 22)	4
4	Three links	$\bar{L}_1$, $\bar{L}_2$, $\bar{L}_4$	(7, 12, 24, 25)	8

Projection Kinematics of Planar Five-Bar Mechanisms



Projection Kinematic Equations

$$
\begin{cases}
L_1 \cos\beta = \bar{L}_1 \\
L_1 \cos\beta - L_2 \cos\beta \cos\theta_{02} + L_2 \sin\alpha \sin\beta \sin\theta_{02} = \bar{L}_1 + \bar{L}_2 \cos(\pi - \bar{\theta}_2) \\
L_2 \cos\alpha \sin\theta_{02} = \bar{L}_2 \sin(\pi - \bar{\theta}_2) \\
L_1 \cos\beta - L_2 \cos\beta \cos\theta_{02} + L_3 \cos\beta \cos(\theta_{02} + \theta_{03}) + L_2 \sin\alpha \sin\beta \sin\theta_{02} - L_3 \sin\alpha \sin\beta \sin(\theta_{02} + \theta_{03}) \\
= \bar{L}_1 + \bar{L}_2 \cos(\pi - \bar{\theta}_2) + \bar{L}_3 \cos(\bar{\theta}_2 + \bar{\theta}_3), \\
L_2 \cos\alpha \sin\theta_{02} - L_3 \cos\alpha \sin(\theta_{02} + \theta_{03}) = \bar{L}_2 \sin(\pi - \bar{\theta}_2) + \bar{L}_3 \sin(-(\bar{\theta}_2 + \bar{\theta}_3)) \\
L_5 \cos\beta \cos\theta_{01} + L_5 \sin\alpha \sin\beta \sin\theta_{01} = \bar{L}_5 \cos\bar{\theta}_1 \\
L_5 \cos\alpha \sin\theta_{01} = \bar{L}_5 \sin\bar{\theta}_1
\end{cases}
$$

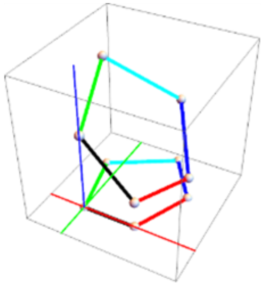
$$
\begin{cases}
\bar{L}_1 - \bar{L}_2 \cos\bar{\theta}_2 + \bar{L}_3 \cos(\bar{\theta}_2 + \bar{\theta}_3) - \bar{L}_5 \cos\bar{\theta}_1 + \bar{L}_4 \cos(\bar{\theta}_1 + \bar{\theta}_5) = 0 \\
\bar{L}_2 \sin\bar{\theta}_2 - \bar{L}_3 \sin(\bar{\theta}_2 + \bar{\theta}_3) - \bar{L}_5 \sin\bar{\theta}_1 + \bar{L}_4 \sin(\bar{\theta}_1 + \bar{\theta}_5) = 0
\end{cases}
$$

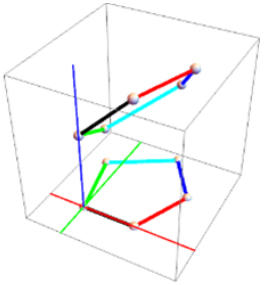
$$
|\mathbf{C}_0 - \mathbf{D}_0| - L_4{}^2 = 0
$$

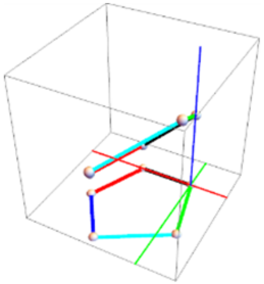
Vector Loop Equation

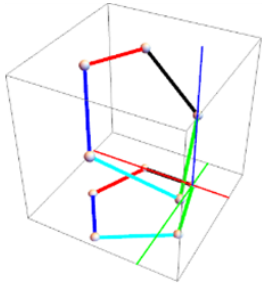
Numerical Example of Planar Five-Bar Mechanisms

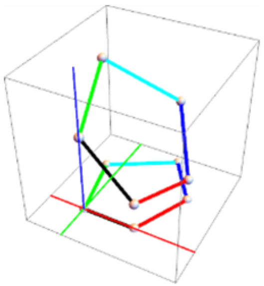
Initial configuration	L_1=2, L_2=5/3, L_3=2, L_4=7/3, θ_{01}=120°, θ_{02}=110°, θ_{03}=110.119°; α = 15°, β=45°
Projected configuration	$\bar{L}_1$=1.414, $\bar{L}_2$=1.663, $\bar{L}_3$=1.505, $\bar{L}_4$=1.945, $\bar{L}_5$=1.804; $\bar{\theta}_1$=103.126°, $\bar{\theta}_2$=114.509°, $\bar{\theta}_3$=121.303°°, $\bar{\theta}_4$=93.22°, $\bar{\theta}_5$=107.842°;

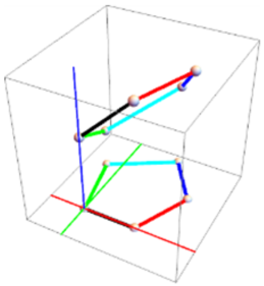

(120°, 110°, 110.119°,15°,45°)

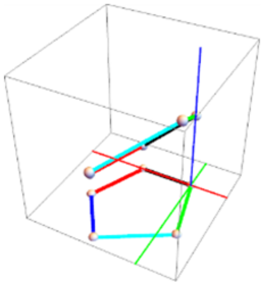

(120°, 110°, 110.119°,-15°,-45°)

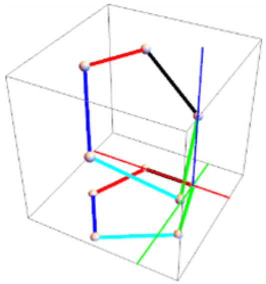

(120°, 110°, 110.119°,-165°,135°)


(120°, 110°, 110.119°,165°,-135°)


(-120°, -110°, -110.119°,-165°,45°)


(-120°, -110°, -110.119°,165°,-45°)


(-120°, -110°, -110.119°,15°,135°)

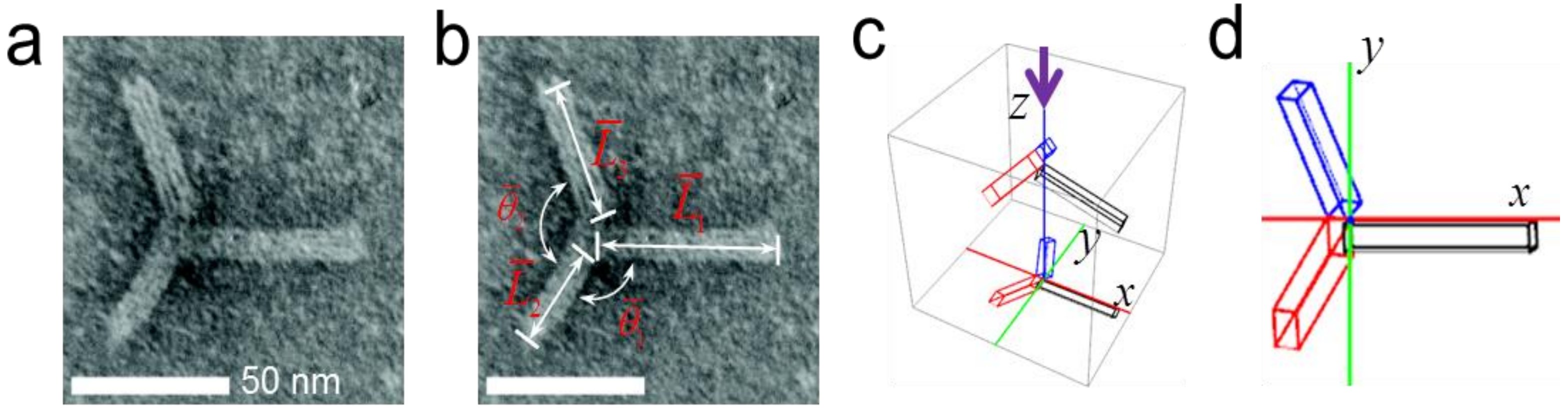

(-120°, -110°, 110.119°,-15°,-135°)

Points in the local reference frame	Points After Rotation	Projecte d points	Measured points from a 2D image
$\mathbf{A}_0 = (L_1, 0, 0)^T$	$\mathbf{A} = [R]\mathbf{A}_0$	$\bar{\mathbf{A}} = [P]\mathbf{A}$	$\bar{\mathbf{A}} = (\bar{L}_1, 0, 0)^T$
$\mathbf{B}_0 = \mathbf{A}_0 +$ $L_2(\cos(\pi - \theta_{02}), \sin(\pi - \theta_{02}), 0)^T$	$\mathbf{B} = [R]\mathbf{B}_0$	$\bar{\mathbf{B}} = [P]\mathbf{B}$	$\bar{\mathbf{B}} = \bar{\mathbf{A}} +$ $(\bar{L}_2 \cos(\pi - \bar{\theta}_2), \bar{L}_2 \sin(\pi - \bar{\theta}_2), 0)$
$\mathbf{C}_0 = \mathbf{B}_0 +$ $(L_3 \cos(\theta_{02} + \theta_{03}), L_3 \sin(-(\theta_{02} + \theta_{03})),$	$\mathbf{C} = [R]\mathbf{C}_0$	$\bar{\mathbf{C}} = [P]\mathbf{C}$	$\bar{\mathbf{C}} = \bar{\mathbf{B}} +$ $(\bar{L}_3 \cos(\bar{\theta}_2 + \bar{\theta}_3), \bar{L}_3 \sin(-(\bar{\theta}_2 + \bar{\theta}_3)$
$\mathbf{D}_0 = (L_5 \cos\theta_{01}, L_5 \sin\theta_{01}, 0)^T$	$\mathbf{D} = [R]\mathbf{D}_0$	$\bar{\mathbf{D}} = [P]\mathbf{D}$	$\bar{\mathbf{D}} = (\bar{L}_5 \cos\bar{\theta}_1, \bar{L}_5 \sin\bar{\theta}_1, 0)^T$

Eight configurations with 5 measurement angles and five measured link length

DNA Origami Universal Joints

- **Universal Joints**



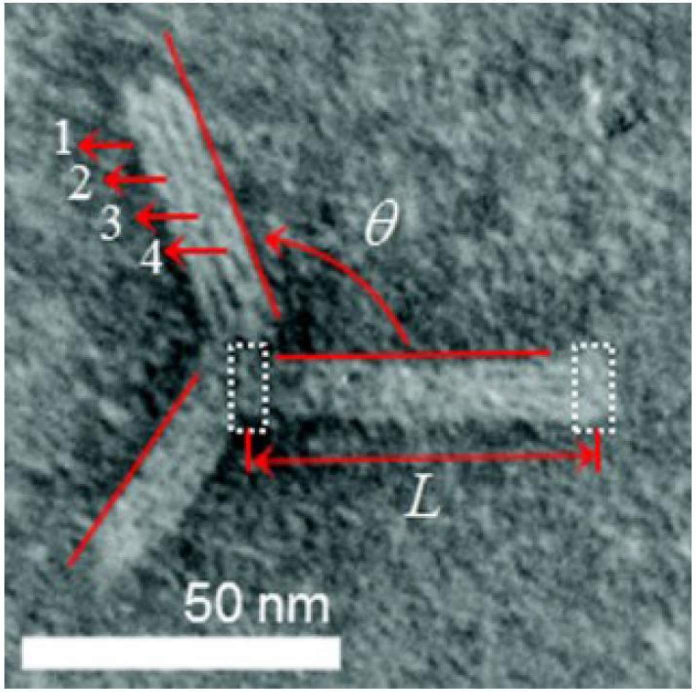
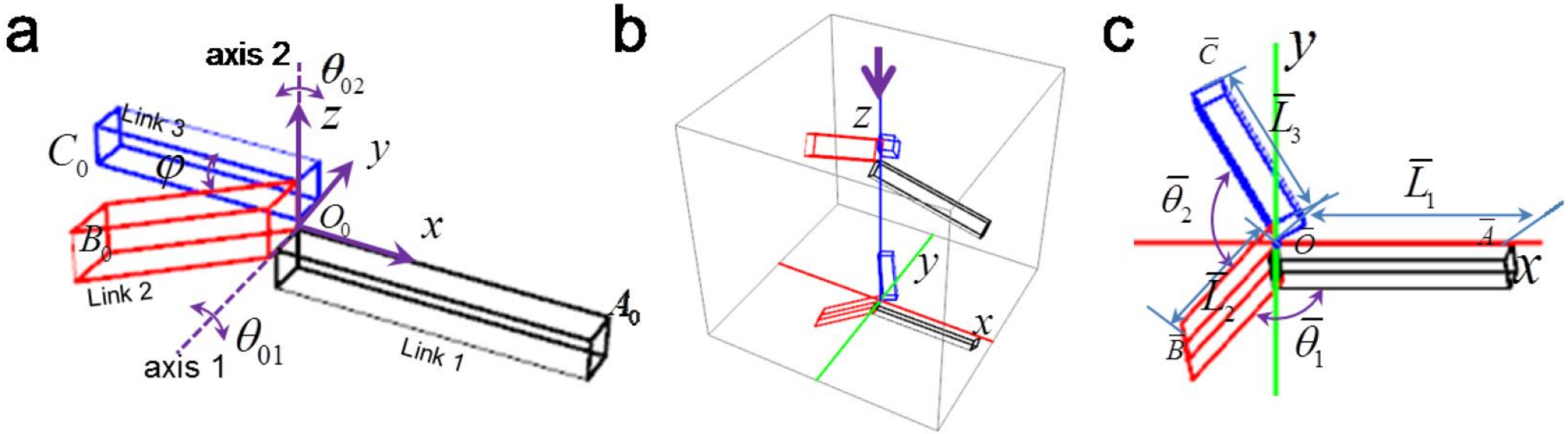


Table 5.12: Initial, rotated, projected and measured points that define the structure of quasi universal joint

True configuration points	Points after rotation	Projected points	Measured points from a 2D image
$\mathbf{A}_0 = (L_1, 0, 0)^T$	$\mathbf{A} = [R]\mathbf{A}_0$	$\bar{\mathbf{A}} = [P]\mathbf{A}$	$\bar{\mathbf{A}} = (\bar{L}_1, 0, 0)^T$
$\mathbf{B}_0 = (-L_2\cos\varphi, -L_2\sin\varphi, 0)^T$	$\mathbf{B} = [R][Y(\theta_{01})]\mathbf{B}_0$	$\bar{\mathbf{B}} = [P]\mathbf{B}$	$\bar{\mathbf{B}} = (\bar{L}_2\cos(-\bar{\theta}_1), \bar{L}_2\sin(-\bar{\theta}_1), 0)^T$
$\mathbf{C}_0 = (-L_3, 0, \Delta h)^T$	$\mathbf{C} = [R][Y(\theta_{01})][Z(\theta_{02})]\mathbf{C}_0$	$\bar{\mathbf{C}} = [P]\mathbf{C}$	$\bar{\mathbf{C}} = (\bar{L}_3\cos(-\bar{\theta}_1 - \bar{\theta}_2), \bar{L}_3\sin(-\bar{\theta}_1 - \bar{\theta}_2), 0)^T$

Note: Δh is the design variable that is shown in Figure 5.12.

DNA Origami Universal Joints



Projection kinematic equations

$$
\begin{cases}
L_1 \cos\beta = \bar{L}_1 \\
L_2(-\cos\beta\cos\theta_{01}\cos\varphi + \cos\alpha\sin\beta\sin\theta_{01}\cos\varphi - \sin\alpha\sin\beta\sin\varphi) = \bar{L}_2\cos\bar{\theta}_1 \\
L_2(\sin\alpha\sin\theta_{01}\cos\varphi + \cos\alpha\sin\varphi) = \bar{L}_2\sin\bar{\theta}_1 \\
L_3(-\cos\beta\cos\theta_{01}\cos\theta_{02} + \cos\alpha\sin\beta\sin\theta_{01}\cos\theta_{02} - \sin\alpha\sin\beta\sin\theta_{02}) = \bar{L}_3\cos(-\bar{\theta}_1 - \bar{\theta}_2) \\
L_3(-\sin\alpha\sin\theta_{01}\cos\theta_{02} - \cos\alpha\sin\theta_{02}) = \bar{L}_3\sin(-\bar{\theta}_1 - \bar{\theta}_2)
\end{cases}
$$

Unknowns

α, β, θ_{01} and θ_{02}

Measurements

$\bar{\theta}_1$, $\bar{\theta}_2$, $\bar{L}_1$, and $\bar{L}_2$

Pick four equations to solve four uknowns.
Use extra measurement data to pick the true solutions

DNA Origami Bennett Mechanisms

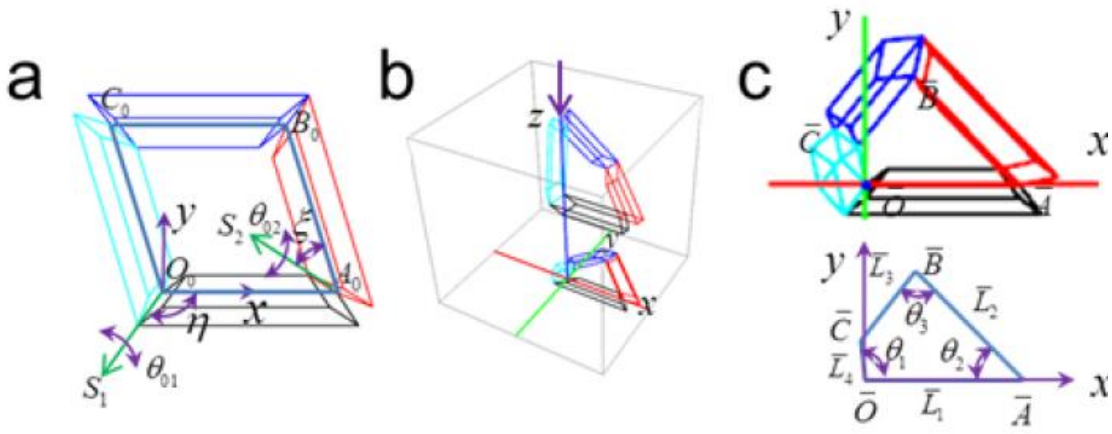


Table 5.14: Initial, rotated, projected and measured points that define the structure of DNA origami Bennett linkage

Points in the local reference frame	Points after rotation	Projected points	Measured points
$\mathbf{A}_0=(L,0,0)^T$	$\mathbf{A}=[R]\mathbf{A}_0$	$\bar{\mathbf{A}}=[P]\mathbf{A}$ $=(A_x,0,0)^T$	$\bar{\mathbf{A}}=(\bar{L}_1,0,0)^T$
$\mathbf{B}_0=\mathbf{A}_0+[e^{\theta_{02}\mathrm{S}_2}]\begin{Bmatrix}-L\\0\\0\end{Bmatrix}$	$\mathbf{B}=[R]\mathbf{B}_0$	$\bar{\mathbf{B}}=[P]\mathbf{B}$ $=(B_x,B_y,0)^T$	$\bar{\mathbf{B}}=\bar{\mathbf{A}}+$ $(-\bar{L}_2\cos(\bar{\theta}_2),\bar{L}_2\sin(\bar{\theta}_2),0)^T$
$\mathbf{C}_0=[e^{\theta_{01}\mathrm{S}_1}]\begin{Bmatrix}L\\0\\0\end{Bmatrix}$	$\mathbf{C}=[R]\mathbf{C}_0$	$\bar{\mathbf{C}}=[P]\mathbf{C}$ $=(C_x,C_y,0)^T$	$\bar{\mathbf{C}}=(\bar{L}_4\cos\bar{\theta}_1,\bar{L}_4\sin\bar{\theta}_1,0)^T$

Note: A_x, B_x, B_y, C_x and C_y are used to represent the coordinates elements because the explicit expressions of them are too complex.

7 projection equations

$$\bar{L}_1+\bar{L}_2\cos(\pi-\bar{\theta}_2)=\bar{L}_4\cos\bar{\theta}_1+\bar{L}_3\cos(\psi_3)$$

$$\bar{L}_2\sin(\pi-\bar{\theta}_2)=\bar{L}_4\sin\bar{\theta}_1+\bar{L}_3\sin(\psi_3),$$

$$A_x=\bar{L}_1,\ B_x=\bar{L}_1+\bar{L}_2\cos(\pi-\bar{\theta}_2),\ B_y=\bar{L}_2\sin(\pi-\bar{\theta}_2),\ C_x=\bar{L}_4\cos\bar{\theta}_1,\ C_y=\bar{L}_4\sin\bar{\theta}_1$$

1 vector loop equation

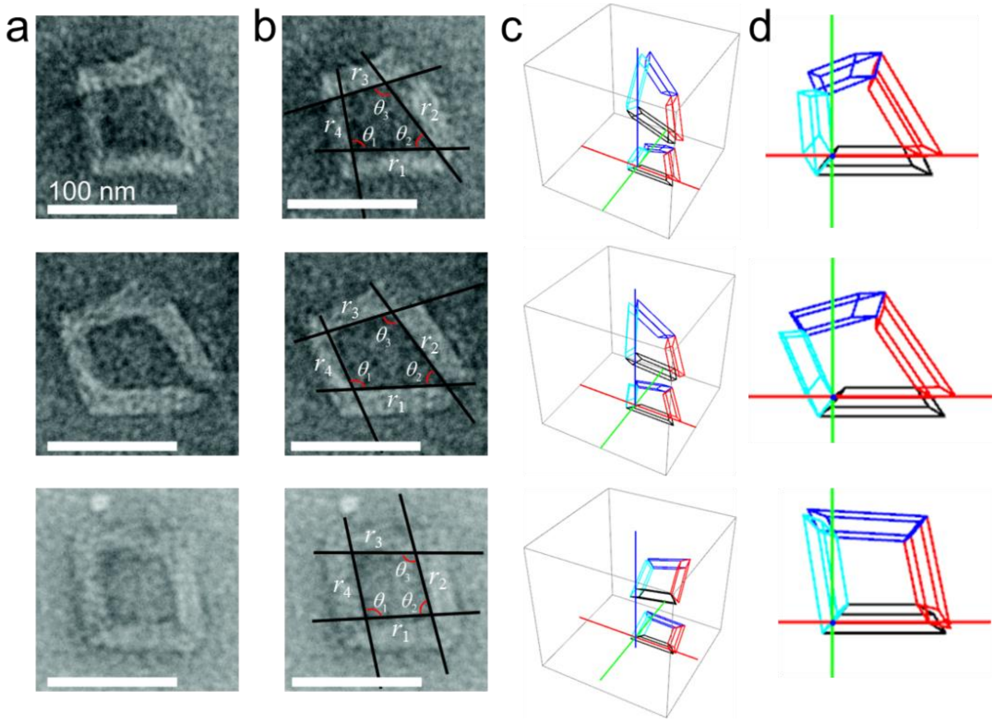
$$(\mathbf{B}_0-\mathbf{C}_0)\bullet(\mathbf{B}_0-\mathbf{C}_0)-L^2=0;$$

DNA Origami Bennett Mechanisms

- **First eliminate** $\bar{L}_1$, $\bar{L}_2$, $\bar{L}_3$ and $\bar{L}_4$ **from four equations. They are linear in equations.**

- **Solve other four unknowns using the other four quations**

θ_{01}, θ_{02}, α and β



Sift Out Extraneous Solutions

- Use extra measurement data to sift out extraneous solutions
- Rank all possible configurations by errors

Table A3

Measurements, projection kinematics solution and projection results from the model, errors of link lengths between the experiment measurements and projection kinematics analysis of DNA origami Bennett linkage (examples 1, 2, and 3 corresponding to the three examples in Fig. 13 from top to bottom respectively), unit for length: nm.

Example	Measurements	Solutions	Projection results from the model	Average error of the link length $\delta = (\sum_{i=1}^{4} \vert\bar{L}_i - \bar{L}_{i-P}\vert/\bar{L}_i)/4)$
1	$\bar{\theta}_1$=101.05°, $\bar{\theta}_2$=52.52°, $\bar{\theta}_3$=110.19°, $\bar{L}_1$=31.64, $\bar{L}_2$=31.20, $\bar{L}_3$=16.93, $\bar{L}_4$=19.71	θ_{01}=155.85°, θ_{02}=67.25°, α=-2.33°, β=22.24°	$\bar{\theta}_{1-P}$=101.05°, $\bar{\theta}_{2-P}$=52.52°, $\bar{\theta}_{3-P}$=110.19°, $\bar{L}_{1-P}$=25.73, $\bar{L}_{2-P}$=26.50, $\bar{L}_{3-P}$=17.55, $\bar{L}_{4-P}$=14.58,	15.9%
2	$\bar{\theta}_1$=113.25°, $\bar{\theta}_2$=55.48°, $\bar{\theta}_3$=109.18°, $\bar{L}_1$=35.29, $\bar{L}_2$=35.03, $\bar{L}_3$=24.11, $\bar{L}_4$=25.89	θ_{01}=154.88°, θ_{02}=66.93°, α=-2.47°, β=5.74°	$\bar{\theta}_{1-P}$=122.20°, $\bar{\theta}_{2-P}$=123.49°, $\bar{\theta}_{3-P}$=123.49°, $\bar{L}_{1-P}$=27.66, $\bar{L}_{2-P}$=27.48, $\bar{L}_{3-P}$=18.82, $\bar{L}_{4-P}$=20.24	21.7%
3	$\bar{\theta}_1$= 99.138°, $\bar{\theta}_2$= 78.27°, $\bar{\theta}_3$= 105.027°, $\bar{L}_1$=25.89, $\bar{L}_2$=25.07, $\bar{L}_3$=24.59, $\bar{L}_4$=25.67	θ_{01}=-124.83°, θ_{02}=104.99°, α=32.41°, β=-9.57°	$\bar{\theta}_{1-P}$= 99.138°, $\bar{\theta}_{2-P}$=78.27°, $\bar{\theta}_{3-P}$=105.027°, $\bar{L}_{1-P}$=27.41, $\bar{L}_{2-P}$=25.38, $\bar{L}_{3-P}$=25.42, $\bar{L}_{4-P}$=27.90	4.8%

Media Reports on DNA Origami Mechanisms

- Article “The Rise of DNA Nanorobots”, Su and Castro, ASME Mechanical Engineering Magazine, 08/2016. Selected as **one of 15 best nanotechnology feature articles since 2010**, The Nanoscale Frontier of ASME, 2016

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- Article “Programmable Motion of DNA origami mechanisms”. (PNAS) 112, no. 3 (January 20, 2015): 713–18. Selected media reports:

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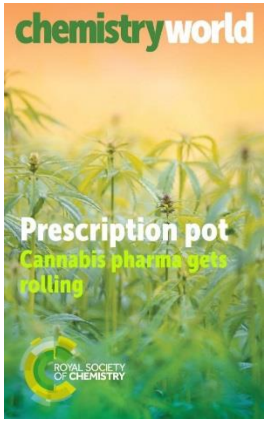

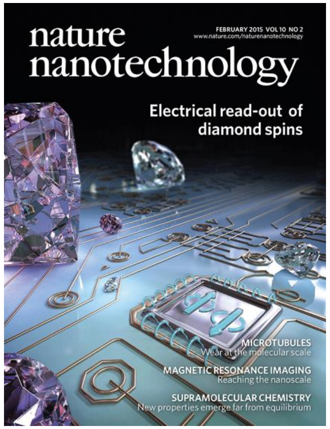

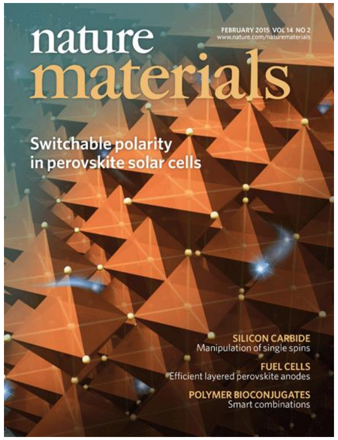

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  - Dr. Gary Ren